

## Thermodynamics of elementary excitations in artificial magnetic square ice

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2012 New J. Phys. 14 015008

(<http://iopscience.iop.org/1367-2630/14/1/015008>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 200.235.132.125

The article was downloaded on 31/01/2012 at 16:39

Please note that [terms and conditions apply](#).

## Thermodynamics of elementary excitations in artificial magnetic square ice

R C Silva, F S Nascimento, L A S Mól, W A Moura-Melo  
and A R Pereira<sup>1</sup>

Departamento de Física, Universidade Federal de Viçosa,  
Viçosa, 36570-000 Minas Gerais, Brazil  
E-mail: [apereira@ufv.br](mailto:apereira@ufv.br)

*New Journal of Physics* **14** (2012) 015008 (14pp)

Received 11 October 2011

Published 31 January 2012

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/14/1/015008

**Abstract.** We investigate the thermodynamics of artificial square spin ice systems assuming only dipolar interactions among the islands that compose the array. Emphasis is given to the effects of temperature on elementary excitations (magnetic monopoles and their strings). By using Monte Carlo techniques we calculate the specific heat, the density of poles and their average separation as functions of temperature. The specific heat and average separation between monopoles with opposite charges exhibit a sharp peak and a local maximum, respectively, at the same temperature,  $T_p \approx 7.2D/k_B$  (here,  $D$  is the strength of the dipolar interaction and  $k_B$  the Boltzmann constant). When the lattice size is increased, the amplitude of these features also increases but very slowly. Really, the specific heat and the maximum of the average separation  $d_{\max}$  between oppositely charged monopoles increase logarithmically with system size, indicating that completely isolated charges could be found only at the thermodynamic limit. In general, the results obtained here suggest that, for temperatures  $T \geq T_p$ , these systems may exhibit a phase with separated monopoles, although the quantity  $d_{\max}$  should not be larger than a few lattice spacings for viable artificial materials.

<sup>1</sup> Author to whom any correspondence should be addressed.

**Contents**

<b>1. Introduction</b>	<b>2</b>
<b>2. The model and outlook</b>	<b>3</b>
<b>3. Results</b>	<b>6</b>
<b>4. Discussion</b>	<b>9</b>
<b>Acknowledgments</b>	<b>13</b>
<b>References</b>	<b>13</b>

**1. Introduction**

New methods of exploring geometric frustrations in magnetic systems have recently been developed. Such methods consist in creating arrays of nanomagnets designed to resemble the disordered magnetic state known as spin ice. They are essentially composed of lithographically defined two-dimensional (2D) ferromagnetic nanostructures (elongated permalloy nanoparticles) with single-domain elements organized in diverse types of geometries (square lattice [1], hexagonal, brickwork [2], kagome [3, 4], etc). Since their geometries are determined lithographically, lattice symmetry and topology can be directly controlled, allowing experimental investigation of a large set of important theoretical models of statistical physics [5]. These artificial magnetic compounds have the potential for increasing our understanding of disordered matter and may also lead to new technologies. Therefore, artificial spin ices are the object of intensive theoretical and experimental investigations [1–4, 6–14].

The trouble is that, in artificial spin ice patterns, the magnetization is unaffected by thermal fluctuations because the magnetic islands contain a large number of spins. Despite the fact that the moment configuration is athermal, these artificial materials can be described through an effective thermodynamics formalism [15, 16]; in addition, some works have introduced a predictive notion of effective temperature [7, 16]. For instance, an external drive, in the form of an agitating magnetic field, behaves as a thermal bath and controls the temperature [7, 16]. Alternatively, this problem was addressed very recently by using a material with an ordering temperature near room temperature [17]; such an experimental work on a square lattice in an external magnetic field confirms a dynamical ‘pre-melting’ of the artificial spin ice structure at a temperature well below the intrinsic ordering temperature of the island material, creating a spin ice array that has real thermal dynamics of the artificial spins over an extended temperature range [17]. These findings and other future possibilities make it evident that a more detailed analysis of the effects of thermal fluctuations on a lower-dimensional spin ice material should be of great interest for a better understanding of these frustrated systems. In particular, it would also be important to know the role of elementary excitations in the thermodynamic properties of artificial magnetic ices.

The main aim of this work is to perform such an investigation. We are interested in the temperature effect on the excitations (‘magnetic monopole defects’ and their strings). Actually, since the prediction of monopoles in the usual 3D spin ice materials [18] and their experimental detection [19–23], the search for these objects in artificial compounds has become an important issue [3, 8, 10, 11]. The possible existence of these excitations in artificial and controllable systems is of great interest because they could be studied at room temperature and, more important, they could be directly observed with modern experimental techniques. Curiously,

in the case of artificial systems, while the square lattice was the first to be produced [1], the direct observation of magnetic monopole defects and their motion was first accomplished in a kagome geometry [3]. Still, in this kagome lattice, a direct, real-space observation of the interplay between strings and monopoles was reported by Mengotti *et al* [4]. For a square lattice, the direct observation of such excitations came only afterwards because there was a primary experimental problem: until last year, none of these systems had achieved its ground state through thermodynamic equilibrium [13]. Despite predictions [6, 8, 9], until recently, studies have not shown a long-range ordered configuration, perhaps because researchers have used only non-thermal methods to randomize the array. This problem was experimentally solved by Morgan *et al* [10]. These authors have reported that by allowing the magnetic islands to interact as they are gradually formed at room temperature, the artificial square spin ice can be effectively thermalized, allowing it to find its predicted ground state very closely; thus, they could also identify the small departures from the ground state as elementary excitations of the system, at frequencies that follow a Boltzmann law. Subsequently, magnetic force microscopy images of a large number of isolated excitations with their string shapes and corresponding moment flip maps were described in square lattices [10]. Therefore, the experimental results considering magnetic artificial square ices obtained in [10] (which demonstrates the thermal ground-state ordering and the elementary excitations) and [17] (which achieves a thermodynamic melting transition by using a material with ordering temperature near room temperature) lead us to think that more progress in the development of such arrays may become available in the near future, establishing opportunities to experimentally elucidate their real thermodynamics.

## 2. The model and outlook

Here, we consider an arrangement of dipoles similar to that experimentally investigated in [1]. In our approach, however, the magnetic moment ('spin') of the island is replaced by an Ising-like point dipole at its center. In this approach, the internal degrees of freedom of each island are not being considered, as well as higher-order interactions. We expect that this simplification does not significantly change the main physical properties of the system. As shown in [24], if the lattice spacing is about twice as large as the island's longest axis, the effect of higher order interactions is negligible. For smaller lattice spacings the effect of higher order interactions is to give more stability to the lowest energy states. In this way one may expect that as the island size increases, approaching the lattice spacing, the ground state should be more robust and the appearance of excitations would cost more energy. While the consideration of the internal degrees of freedom would reduce the energy scale, the consideration of higher order interactions would increase it, but none of them are expected to change the physical picture discussed here. Thus, in our approach, at each site  $(x_i, y_i)$  of the square lattice two spin variables are defined:  $\vec{S}_{x(i)}$  with components  $S_x = \pm 1, S_y = 0, S_z = 0$  located at  $\vec{r}_x = (x_i + 1/2, y_i)$  and  $\vec{S}_{y(i)}$  with components  $S_x = 0, S_y = \pm 1, S_z = 0$  located at  $\vec{r}_y = (x_i, y_i + 1/2)$ . Therefore, in a lattice of volume  $L^2 = l^2 a^2$  ( $a$  is the lattice spacing) one gets  $2 \times l^2$  spins. Representing the spins of the islands by  $\vec{S}_i$ , which can assume either  $S_{x(i)}$  or  $S_{y(i)}$ , then the artificial spin ice is described by the following Hamiltonian:

$$H_{SI} = Da^3 \sum_{i \neq j} \left[ \frac{\vec{S}_i \cdot \vec{S}_j}{r_{ij}^3} - \frac{3(\vec{S}_i \cdot \vec{r}_{ij})(\vec{S}_j \cdot \vec{r}_{ij})}{r_{ij}^5} \right], \quad (1)$$

where  $D = \mu_0\mu^2/4\pi a^3$  is the coupling constant of the dipolar interaction. We use standard Monte Carlo techniques to obtain thermodynamic averages of the system defined by Hamiltonian (1). Periodic boundary conditions were implemented by means of the Ewald summation [25, 26], used here to avoid spurious results brought about by the use of a cutoff radius [27]. Our Monte Carlo procedure comprises a combination of single-spin flips and loop moves [28], where all spins contained in a closed random loop are flipped according to the Metropolis prescription. In our scheme one Monte Carlo step (MCS) consists of  $2 \times l^2$  single-spin flips and  $0.7 \times l^2$  worm moves. Usually,  $10^4$  MCS were shown to be sufficient to reach equilibrium configurations and we have used  $10^5$  configurations to get thermodynamic averages.

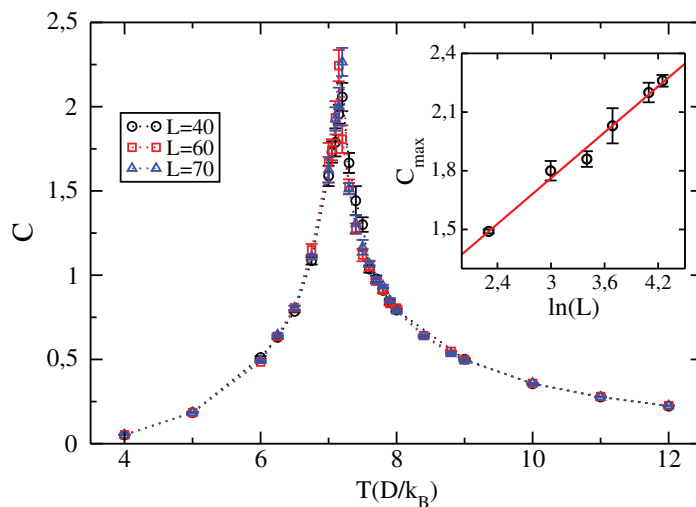
Before presenting the Monte Carlo calculations, it would be interesting to remark on some previous results [8, 9, 11] and some expectations for these arrays. The ground-state configuration of the system in a square lattice is twofold degenerate. If one considers the vorticity in each plaquette, assigning a variable  $\sigma = +1$  and  $-1$  to clockwise and anticlockwise vorticities, respectively, the ground state looks like a checkerboard, with an antiferromagnetic (AF) arrangement of the  $\sigma$  variable [8, 10]. Of course, the ground state clearly obeys the ice rule (two spins point inward and two point outward in each vertex), but with configurations of topology 1 (in 2D, there are two topologies that obey the ice rule. However, they are not degenerate and topology 2 is more energetic than topology 1; see [1, 8] for more details). The most elementary excitation is related to the inversion of a single spin (dipole) to generate a localized pair of defects. This is the 3-in, 1-out state in a particular vertex and the 3-out, 1-in state in its adjacent vertex. In principle, these defects could be separated without further violation of the ice rule. Indeed, in our previous papers [8, 9], we have numerically shown that these defects behave as a monopole pair since their interaction follows a  $d = 3$  Coulomb law  $q/R$ , where  $q$  measures the strength of the interaction and  $R$  is the distance between the poles. However, we have also pointed out that an isolated monopole should be hard to see as effective low-energy degrees of freedom in the 2D square spin ice, because the background AF order in the ground state confines them [8], since the ice rule is not degenerate in 2D. Actually, in 2D, there are additional excitations not present in the usual 3D spin ice [18], namely energetic 1D strings of dipoles (resultant spins at each vertex along a line of adjacent vertices) that terminate in monopoles with opposite charges. Such string excitations could be seen as lines which pass by adjacent vertices that obey the ice rule but sustaining topology 2 (instead of topology 1) and hence they cost an energy equal to  $b$  times their length  $X$ , where  $b$  is the string tension. When the temperature  $T$  of the system is near absolute zero, the shortest path length connecting the monopoles gives the potential energy. The most general expression for the total cost of a pair of monopoles separated by a distance  $R$  is the sum of the usual Coulombic term roughly equal to  $q/R$  and a term roughly equal to  $bX$  resulting from the string joining the monopoles (there is, of course, also a constant term associated with the creation energy of a pair). Note that there is not a unique identification of a given path connecting the ends (monopoles) of the excitation. It is explicitly considered in the fact that the energy is proportional to  $X$ , which can assume different values for a given  $R$ . For a sufficiently long string, the string energy is completely dominant; for a short string the Coulomb interaction may have some importance if the size of the end-point monopoles is even smaller (as always occurs for these systems). With the above features, these excitations are, to some extent, more similar to Nambu monopoles [29] than to Dirac monopoles. Really, as Nambu suggested, for a modified Dirac monopole theory, the string connecting monopoles has energy and is oriented, having a sense of polarization [29].

In the artificial square ices, the ordering causes an anisotropy in the system making the monopoles interaction highly dependent on the direction in which the monopoles are separated in the crystal plane [9]. This anisotropy is manifested in both the Coulomb and linear terms of the potential in such a way that we explicitly write [9]

$$V(R) = q(\phi)/R + b(\phi)X + c, \quad (2)$$

where  $\phi$  is the angle that the line joining the monopole defects makes with the  $x$ -axis of the array. Numerically, for instance,  $q(0) \approx -3.88Da$ ,  $b(0) \approx 9.8D/a$ , while  $q(\pi/3) \approx -4.1Da$ ,  $b(\pi/3) \approx 10.1D/a$ . The constant  $c \approx 23D$ , associated with the pair creation energy [9] ( $E_c \approx 29D$ ), is independent of  $\phi$ . Similar results can be found in the experimental work for the square lattice. Indeed, in [10], the authors have found that, at a temperature  $T$ , these excitations arise in the system according to the Boltzmann law  $\sim \exp(-\beta V(R))$  with  $b \approx 10D/a$ ,  $V(a) = E_c \approx 30D$  and  $\beta = 1/k_B T$ , where  $k_B$  is the Boltzmann constant. They have also classified the elementary excitations by the number of flipped spins (given by  $n$ ) and a mnemonic character for shape. The three most observed defects are represented by 1 (a single pair with charges separated by only one lattice spacing) followed by  $2L$  (a pair with  $n = 2$  with the shape of  $L$ ) and  $4O$  (an isolated string loop with no charges and having  $n = 4$  flipped spins) [10]. Curiously, the second excited state should be  $4O$  since its energy is smaller than the energy of the  $2L$  defect.

In principle, for the thermodynamics of these systems, the following argument should be valid: at low temperatures, there is insufficient thermal energy to create long strings (with length  $X$  larger than one lattice spacing) and so the monopoles (with opposite charges) are bound together tightly in pairs. On the other hand, as the temperature is increased, the average separation between the constituents of a pair should also increase, which means that larger strings may become present in the system. Of course, there are several ways of connecting two monopoles by a string of length  $X$ . Therefore, considering states with  $X \gg R$ , we remember then that the number of configurations for the  $m$ -step self-avoiding random walk is  $N = \delta^m$ , where  $\delta$  is a constant and equal to 3 for a 2D square lattice. For a string with a sufficiently large  $X$ ,  $N$  is well approximated by the random walk result and one obtains  $N \simeq \delta^{X/a}$ . So the entropy of strings is proportional to  $X$ , i.e. the many possible ways of connecting two monopoles with a string give rise to a string configurational entropy proportional to  $X$ . Crudely speaking, then, the string free energy  $F = [b - (\ln 3)k_B T/a]X$  will imply an effective string tension  $[b - (\ln 3)k_B T/a]$  which is positive in the low temperature region and the monopoles are completely confined. Above a certain temperature, it becomes negative; namely, the string loses its tension. The tension decreases like  $[b - (\ln 3)k_B T/a]$  with increasing  $T$ , vanishing at some critical temperature  $k_B T_c \approx ba/\ln 3$ . Using the average value for the string tension in equation (2), i.e.  $b \approx 10D/a$ , we then estimate that  $k_B T_c \approx 9.1D$ . Of course, these theoretical arguments always overestimate the critical temperature. Although this picture leads to rich physics for this system, predicting free magnetic monopoles and a phase transition, things may be a little more complicated. Really, additionally to the entropic effect discussed just above, there is another entropic contribution which manifests against monopole separation; the monopoles should become close together because it would provide more ways of arranging the surrounding dipoles in the lattice. Such an effect introduces a 2D Coulombic interaction between the poles, which is proportional to  $T$  (i.e.  $V_s = T \ln(R/a)$ ). If the temperature at which the string loses its tension is high enough, of the order of  $9.1D$  as estimated, then, around this value of  $T$ , the confining potential  $V_s$  must be very strong, possibly preventing



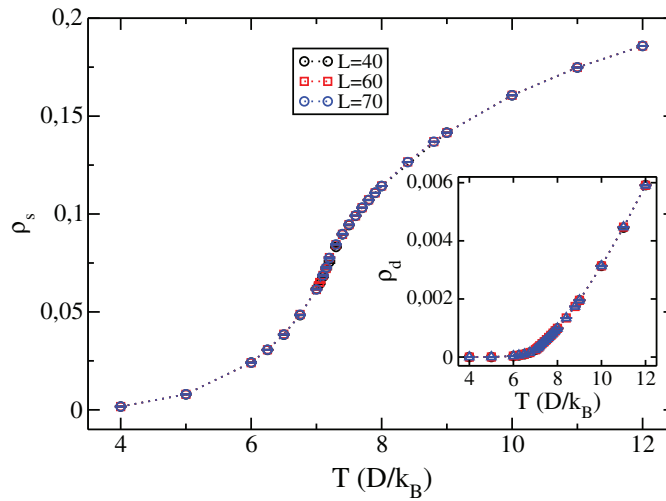
**Figure 1.** Specific heat as a function of temperature. The figure exhibits a sharp peak at a temperature  $T_p \sim 7.2D/k_B$ , at which the amplitude increases very slowly with system size  $L$ . Inset: the specific heat peak diverges logarithmically with system size  $L$ .

freedom of the poles. With all these expectations, it would be important to investigate how the elementary excitations behave as a function of temperature. Our calculations are a first step in this direction.

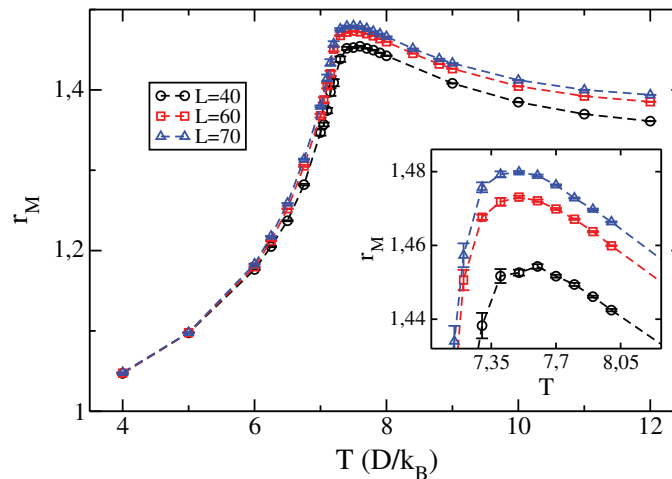
### 3. Results

We now present the results of Monte Carlo simulations. The calculations shown here are for lattices with sizes 10, 20, 30, 40, 50, 60 and 70 lattice spacings, but in all figures we present only the results for lattice sizes 40, 60 and 70. We start by presenting the results for the specific heat (see figure 1). We note that, for all lattice sizes studied, the specific heat exhibits a sharp feature at a temperature  $T_p$  approximately equal to  $7.2D/k_B$ . Indeed, the position of this peak does not seem to move as the lattice size  $L$  is varied. On the other hand, its amplitude  $C_{\max}$  increases very slowly as  $L$  increases. In the inset of figure 1, we show how  $C_{\max}$  behaves with  $L$ . Therefore, with the obtained data we expect a logarithmic divergence of the specific heat in the thermodynamic limit. We also analyzed the pair density and the average separation between monopoles with opposite charges as a function of  $T$ . It is useful here to distinguish two types of monopoles: the less energetic ones in which the spins (in a vertex) are in the 3-in, 1-out or 3-out, 1-in states (here referred to as unit-charged monopoles) and the most energetic ones in which the spins are in the 4-in or 4-out states (doubly charged monopoles). Figure 2 shows the density of pairs containing monopoles with unitary charge ( $\rho_S$ ) and also the density of pairs containing doubly charged monopoles ( $\rho_D$ , see the inset). They are calculated as one-half of the thermodynamic average of the absolute value of the charge ( $\pm 1$ ) and ( $\pm 2$ ), respectively, summed over the lattice. For both cases, the density increases monotonically up to a maximum value achieved in the high-temperature limit.

The size of the monopole pairs constitutes an internal degree of freedom, since the energy of a pair depends on the distance between the members of the pair. Here we would like to know the average distance  $r_M$  between two opposite poles as a function of temperature. Such a



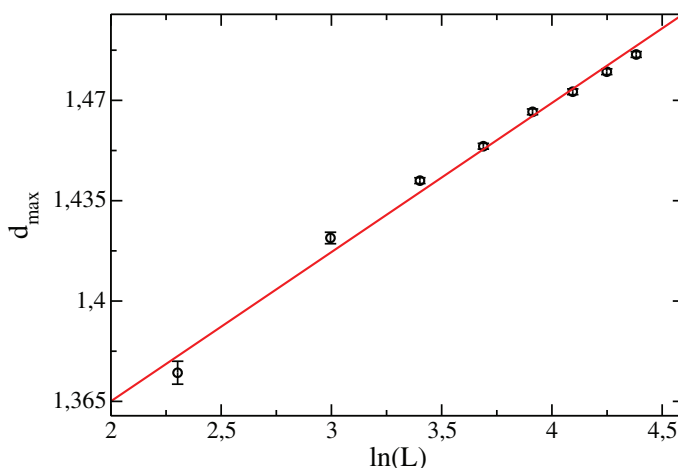
**Figure 2.** The density of pairs of unit-charged monopoles as a function of temperature. Inset: the density of doubly charged monopole pairs.



**Figure 3.** The average separation between charges exhibits a maximum around the same temperature  $T_p$  at which the specific heat has a sharp feature. The inset shows, in more detail, the region around the maximum.

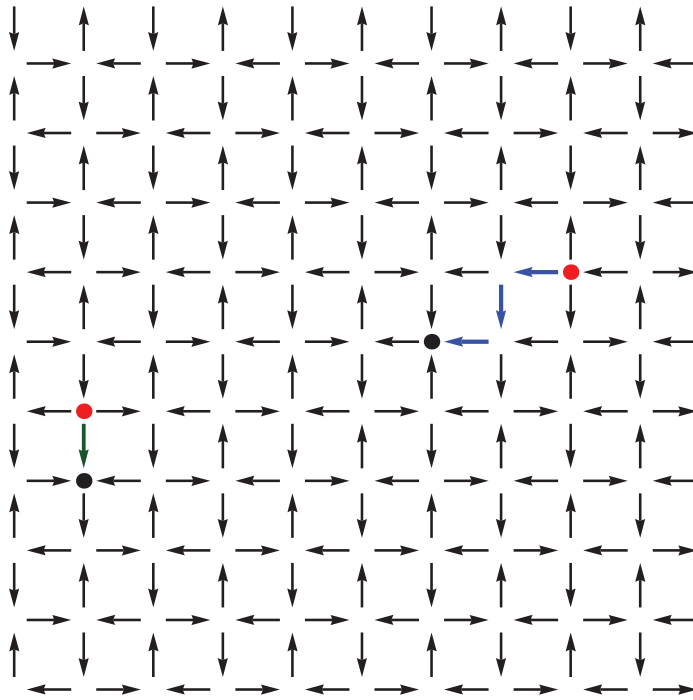
thermodynamic quantity may contain information about the possibility of monopole separation and how they are organized into the system. For this calculation we consider only defects with unitary charges. The grouping of monopoles into pairs is unique as long as the distances between them are smaller than the average distance between the monopoles  $r_M = 1/\sqrt{\rho_s}$ . As the size of the monopole pairs becomes larger than  $r_M$ , one would simply have to redefine the monopole pairs. The average size  $r_M$  of the monopole pairs is calculated by using the method of assignment problems; it deals with the question of how to assign  $n$  items (jobs, students) to  $n$  other items (machines, tasks) [30]. In our case, we would like to assign  $n$  positive charges to  $n$  negative charges for a given configuration in such a way that the sum of distances of all possible pairings be a minimum. The results are shown in figure 3. The average separation has a local maximum at the same temperature  $T_p$  at which the specific heat exhibits a peak ( $\sim 7.2D/k_B$ ). We note that





**Figure 4.** The maximum of the average separation  $d_{\max}$  between opposite charges increases logarithmically with the system size  $L$ .

the amplitude of this maximum increases slowly as the system size increases. Indeed, like the specific heat peak, the maximum in the average separation  $d_{\max}$  also increases logarithmically with the system size  $L$  ( $d_{\max} \propto \ln L$ , see figure 4) and hence one could expect that a certain number of monopoles may be almost isolated for very large arrays. Indeed, in our simulations for temperatures  $T \geq T_p$  considering lattices with  $L \leq 80a$ , we could observe some charges relatively distant from their respective counterparts (separated by distances of the order of  $5a$ ). For instance, we show in figure 5 a distribution of positive (red circles) and negative (black circles) monopoles in a small lattice with  $L = 10a$  observed in our simulations for a temperature  $T = 6.0D/k_B$  (i.e. below  $T_p$ ). Note that there are a very few excitations and all monopoles with opposite charges are coupled by a string, forming pairs. On the other hand, figure 6 shows the same system for a temperature above  $T_p$  ( $T = 7.6D/k_B$ ). In this case, we see that a small number of monopoles are not connected by strings. In principle, they are free, although some of them are not completely isolated (i.e. far away from other opposite poles). Furthermore, we also note that some strings seem to be detached, not terminating in monopoles; there are a few pieces of string dispersed along the system (as stated before, strings could be seen as lines which pass by adjacent vertices that obey the ice rule but sustaining topology 2 rather than topology 1). Of course, these figures exhibit only samples from a large number of data, but most of the data should be similar to the features of figure 5 for the regime of low temperatures and the features of figure 6 for the regime of high temperatures. Things must be clearer in the thermodynamic limit; in this case, some monopoles should become infinitely separated from their counterparts for temperatures  $T \geq 7.2D/k_B$ . However, as the temperature is increased from zero, the monopole pair density grows simultaneously with an increase in pair size (see also figure 2). As the pairs become denser, there is less space to put in new pairs and hence the average pair size  $r_M$  decreases for high temperatures. Really, we observe that, for  $T < T_p$ , the average separation  $r_M$  does not depend on the lattice size  $L$ , while for  $T \geq T_p$ , this quantity has a tiny dependence on  $L$  (at least in the range  $7.2D/k_B < T < 12D/k_B$ ). In this case, it is possible that monopoles may become completely isolated even for high temperatures ( $T > T_p$ ) when  $L \rightarrow \infty$ . This picture for infinite systems corroborates the theoretical expectations for the existence of a phase with free monopoles [8] in large 2D artificial square ices, but the transition temperature ( $\sim 7.2D/k_B$ )



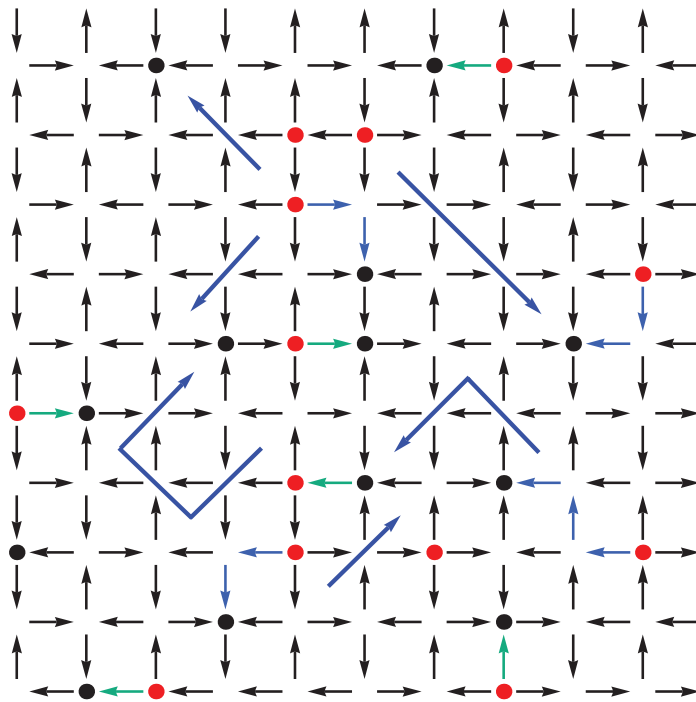
**Figure 5.** Snapshot of a particular configuration of excitations for a temperature  $T = 6.0D/k_B$  in a lattice with  $L = 10a$ . Red and black circles are positive and negative charges, respectively. In general, for all temperatures below  $T_p$ , each monopole is clearly confined to its counterpart by a string (see the blue arrow indicating the direction of the string for the larger pair). Small pairs (i.e. monopoles bound together tightly in pairs) are indicated by a green arrow.

should be a little smaller than the estimated value  $\sim 9.1D/k_B$  discussed earlier (remember that the arguments of energy-entropy, in general, overestimate the correct quantity).

We have also calculated the density of string loops  $4O$ , which is the defect with no charge but having the second lowest energy (second excited state). Like the specific heat and the average separation, the density of defects  $4O$  also displays a feature at  $T_p$  (see figure 7). Note that the string loops  $4O$  almost do not appear in the system for temperatures smaller than  $T_p$ . Indeed, they surge suddenly at  $T_p$  and then, for temperatures above  $T_p$ , their number starts to decrease while the density of monopole pairs starts to increase more appreciable. Figures 8 and 9 show typical distributions of defects  $4O$  in the system for temperatures below and above  $T_p$ , respectively.

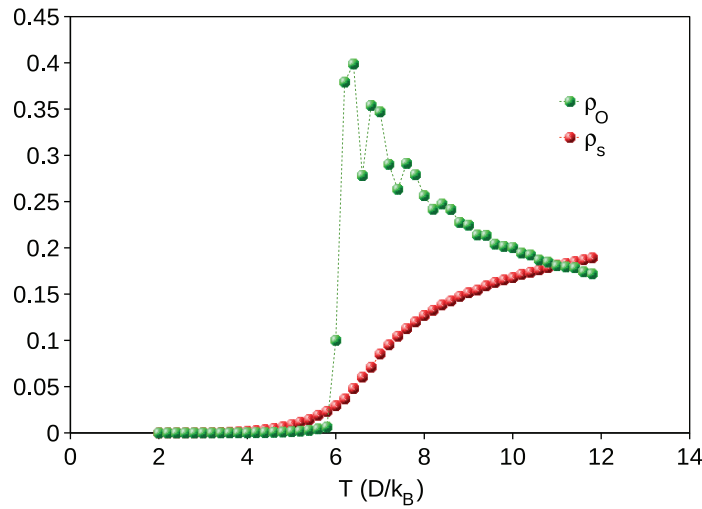
#### 4. Discussion

In summary, assuming the spin–spin interaction to be purely dipole–dipole, we note that, at a temperature  $T_p$ , there is a maximum in the mean separation of opposite monopoles that increases logarithmically with the system size  $L$  ( $d_{\max} \propto \ln L$ ). Hence, the distance between monopoles with opposite charges in the thermodynamic limit ( $L \rightarrow \infty$ ) should diverge weakly, suggesting a possible unbinding of monopole pairs ( $T < T_p$ ) into ‘free’ monopoles ( $T > T_p$ ). However, to the authors’ knowledge, for a finite monopole density there is no diagnostic for

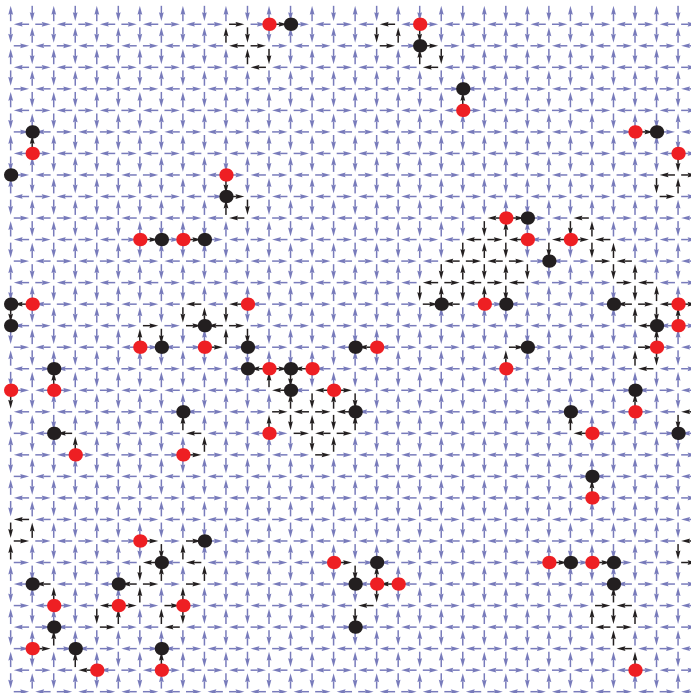


**Figure 6.** Snapshot of a particular configuration of excitations for a temperature  $T = 7.6D/k_B$  in a lattice with  $L = 10a$ . Red and black circles are positive and negative charges, respectively. For a temperature above  $T_p$ , a small number of monopoles do not have a string connecting them to their counterparts and therefore seem to be isolated. There are also some pieces of strings (i.e. 1D regions obeying topology 2, as indicated by blue paths) that do not connect monopoles. Small pairs are indicated by a green arrow.

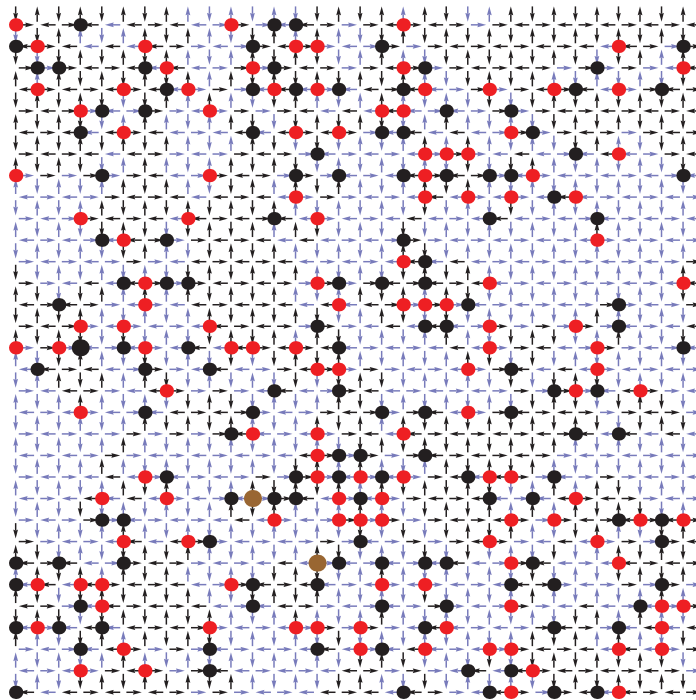
(de)confinement based on a pair distribution function, for reasons analogous to the failure of the Wilson loop (which only knows perimeter laws in the presence of dynamical matter) to diagnose deconfinement in gauge theories. Indeed, from the three approaches that have been used to measure the static potential associated with the breaking of a long flux tube between two quarks in quantum chromodynamics (i.e. correlation of Polyakov loops, variational ansatz and Wilson loops), string breaking has been seen only using the first two methods. On the other hand, the divergence found in  $r_M$  could be understood in two different ways. It may be associated with either a vanishing string tension (which would lead to effectively free poles) or simply the fact that in an order–disorder transition the correlation length (which is the only characteristic length of the system) diverges at the critical temperature. In this case, since the mean distance should be given in terms of the correlation length, it should also diverge. Of course, these two distinct ways of describing the system are closely related. We are faced thus with the question of the existence or not of a phase transition in this system. If there is a phase transition, another question arises: what is its nature? It is worthy of note at this point [31] that, although this system is closely related to the 16-vertex model, for which an exact solution is known, the range and symmetry of the interactions differ and thus we do not expect to observe the same critical behavior. Nevertheless, one point that deserves remark is the possible similarities between this system and the Ising



**Figure 7.** The density of string loops and of pairs with opposite charge as a function of temperature. The density of string loops  $4O$  ( $\rho_O$ ) also exhibits a maximum around the temperature  $T_p \simeq 7.2D$  (green balls). This defect carries no charge and is the second excited state. Just for comparison, the density of pairs with opposite charges ( $\rho_s$ ) is also shown (red balls).



**Figure 8.** A typical configuration of string loops of the type  $4O$  for a temperature below  $T_p$  (here,  $T = 6D/k_B$ ). At  $T_p$ , the number of  $4O$  excitations proliferates in such a way that a percolated cluster seems to be formed. The figure also shows the pairs of monopoles.



**Figure 9.** A typical configuration of string loops of the type  $4O$  for a temperature above  $T_p$  (here,  $T = 8D/k_B$ ). The figure also shows the pairs of monopoles.

model. In the two degenerate ground states, the  $\sigma$  variables, related to the vorticity of each plaquette, can be seen as the spins of an AF Ising model. In the AF Ising model, as the temperature rises, clusters of flipped spins are found in the system and at the critical temperature one can find percolated clusters of spins. If there are some similarities between these systems one may expect thus that the  $4O$  excitations, which can be viewed as flipped  $\sigma$  variables, form clusters at low temperature that percolate at the critical temperature, justifying the increasing number of these excitations at the transition temperature. This picture is corroborated by the logarithmic divergence of the specific heat. Unfortunately, our results are not conclusive about the possibility of a phase transition, and much more work has to be done in order to answer this question. To try to throw some extra light on the topic, we have also performed some calculations restricting the islands interaction to nearest neighbors converging in the same vertex, which would lead to a kind of generalized 2D Ising system with the same ground state. Nevertheless, we found that the vertices with topology 3, in the 3-in/1-out and 3-out/1-in states, remain connected by strings (but now there is no Coulomb interaction anymore). The interaction energy between two opposite vertices in topology 3 (type III vertices) is given by  $b_1 X + c_1$ , where  $b_1 = 26D/a$  and  $c_1 = 34D$ , much bigger than the usual results obtained for the long-range dipolar interaction. Since the string tension persists, the arguments associated with the string configurational entropy should remain valid and again we have the same problem as before (but with different energetics; for instance, the value of the temperature at which the quantities show a maximum changes to  $16D/k_B$ ). Indeed, the specific heat, the average separation between opposite type III vertices, etc, show the same behavior as that found for the system with long-range dipolar interaction (not shown here).

From the practical point of view, the divergence in  $r_M$  in the thermodynamic limit, and thus the phase of large separation among monopoles, should not be expected in finite systems. Due to the slow logarithmic divergence, the extrapolation of our results to a 2D lattice containing Avogadro's number ( $N_a^{2/3} = 10^{16} = 10^8 \times 10^8$ ) of islands will imply  $d_{\max} \sim 2.5a$  only. On the other hand, even with small values for  $d_{\max}$ , some monopoles may become isolated for temperatures near  $7.2D$  (see figure 6). The challenge of building arrays using new materials (with an ordering temperature near room temperature) and/or with reduced island volume and moment (and possibly with larger  $L$ ) should thus be an important issue in technological applications. Indeed, it concerns the excitation evolution in these artificial compounds. These developments may experimentally determine the possibility of monopole dynamics, their lifetimes and so on. For instance, on the basis of only the average separation results, we speculate that near the temperature  $T_p$ , the annihilation process of monopoles (without strings) will more probably occur in small arrays than in large arrays, because the mean separation between such opposite charges increases with system size.

### Acknowledgments

We thank CNPq, FAPEMIG, CAPES and FUNARBE (Brazilian agencies) for financial support. We also thank Professors R Moessner and G M Wysin for a careful reading of the manuscript and for helpful comments.

### References

- [1] Wang R F *et al* 2006 *Nature* **439** 303
- [2] Li J, Ke X, Zhang S, Garand D, Nisoli C, Lammert P, Crespi V H and Schiffer P 2010 *Phys. Rev. B* **81** 092406
- [3] Ladak S, Read D E, Perkins G K, Cohen L F and Brandford W R 2010 *Nat. Phys.* **6** 359
- [4] Mengotti E, Heyderman L J, Rodriguez A F, Nolting F, Hügli R V and Braun H-B 2011 *Nat. Phys.* **7** 68
- [5] Baxter R 1982 *Exactly Solved Models in Statistical Physics* (New York: Academic)
- [6] Möller G and Moessner R 2006 *Phys. Rev. Lett.* **96** 237202
- [7] Ke X, Li J, Nisoli C, Lammert P E, McConville W, Wang R F, Crespi V H and Schiffer P 2008 *Phys. Rev. Lett.* **101** 037205
- [8] Mól L A , Silva R L, Silva R C, Pereira A R, Moura-Melo W A and Costa B V 2009 *J. Appl. Phys.* **106** 063913
- [9] Mól L A S, Moura-Melo W A and Pereira A R 2010 *Phys. Rev. B* **82** 054434
- [10] Morgan J P, Stein A, Langridge S and Marrows C 2011 *Nat. Phys.* **7** 75
- [11] Möller G and Moessner R 2009 *Phys. Rev. B* **80** 140409
- [12] Zabel H, Schumann A, Westphalen W and Remhof A 2009 *Acta Phys. Pol. A* **115** 59
- [13] Budrikis Z, Politi P and Stamps R L 2010 *Phys. Rev. Lett.* **105** 017201
- [14] Libál A, Reichhardt C and Reichhardt C J O 2011 arXiv:1108.3584 [cond.mat]
- [15] Nisoli C, Wang R, Li J, McConville W F, Lammert P E, Schiffer P and Crespi V H 2007 *Phys. Rev. Lett.* **98** 217203
- [16] Nisoli C, Li J, Ke X, Garandi D, Schiffer P and Crespi V H 2010 *Phys. Rev. Lett.* **105** 047205
- [17] Kapaklis V, Arnalds U B, Harman-Clarke A, Papaioannou E Th, Karimipour M, Korelis P, Taroni A, Holdsworth P C W, Bramwell S T and Hjörvarsson B 2011 arXiv:1108.1092v1 [cond.mat]
- [18] Castelnovo C, Moessner R and Sondhi L 2008 *Nature* **451** 42
- [19] Fennell T, Deen P P, Wildes A R, Schmalzl K, Prabhakaran D, Boothroyd A T, Aldus R J, McMorrow D F and Bramwell S T 2009 *Science* **326** 415

- [20] Morris D J P *et al* 2009 *Science* **326** 411
- [21] Bramwell S T, Giblin S R, Calder S, Aldus R, Prabhakaran D and Fennell T 2009 *Nature* **461** 956
- [22] Kadowaki H, Doi N, Aoki Y, Tabata Y, Sato T J, Lynn J W, Matsuhira K and Hiroi Z 2009 *J. Phys. Soc. Japan* **78** 103706
- [23] Jaubert L D C and Holdsworth P C W 2009 *Nat. Phys.* **5** 258
- [24] León A and Pozo J 2008 *J. Magn. Magn. Mat.* **320** 210
- [25] Wang Z and Holm C 2001 *J. Chem. Phys.* **115** 6351
- [26] Weis J-J 2003 *J. Phys.: Condens. Matter* **15** S1471
- [27] Mól L A S and Costa B V 2011 arXiv:1109.1840v1 [cond.mat]
- [28] Barkema G T and Newman M E J 1998 *Phys. Rev. E* **57** 1155
- [29] Nambu Y 1974 *Phys. Rev. D* **10** 4262
- [30] Bukard R, Dell'Amico M and Martello S 2009 *Assignment Problems* (Philadelphia: Society for Industrial and Applied Mathematics)
- [31] Mól LAS, Pereira A R and Moura-Melo W A 2011 *Phys. Lett. A* **375** 2680