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STATISTICAL

Vortices in Low-Dimensional Magnetic Systems

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Abstract Vortices are objects that are important to describe several physical phenomena. There are many examples of such objects in nature as in a large variety of physical situations like in fluid dynamics, superconductivity, magnetism, and biology. Historically, the interest in magnetic vortex-like excitations begun in the 1960s. That interest was mainly associated with an unusual phase-transition phenomenon in two-dimensional magnetic systems. More recently, direct experimental evidence for the existence of magnetic vortex states in nano-disks was found. The interest in such model was renewed due to the possibility of the use of magnetic nano-disks as bit elements in nano-scale memory devices. The goal of this study is to review some key points for the understanding of the vortex behavior and the progress that have been done in the study of vortices in low-dimensional magnetic systems.

Keywords Vortex · Magnetism

1 Introduction

Vortex is a general name for an object that can be thought of as streamlines of circulating fluid flowing around a hole sink (see Fig. 1). There are many examples of such objects in nature as in a large variety of physical situations like in fluid mechanics, superconductivity, magnetism, and many others [1–4]. For example, vortex generators are used in commercial airplanes to improve the quality of the flight [1], in superconductivity vortices are associated with the phase of the superconducting order parameter [3]. In magnetic systems they are believed to be responsible for an exotic phase transition in two-dimensional anisotropic Heisenberg models [5, 6]. Recently, they emerged as candidates to improve miniaturization in magnetic storage devices.

Historically, the interest in magnetic vortex-like excitations in two-dimensional models begun in the 1960s. That interest was mainly associated with the phase transition phenomenon. In one dimension, non-linear excitations called kinks [7–11] are responsible for destroying the order at any finite temperature in short-range potential models. In two dimensions, vortices are the important excitations. Although the Mermin–Wagner theorem [12, 13] demonstrate the impossibility of the existence of an order-disorder transition in models with continuous order parameter, in two dimensions or less, the Heisenberg model with an easy-plane anisotropy still undergoes an unusual phase transition with no true long-range order. There are two interpretations for the existence of this transition. One by Berezinskii, Kosterlitz, and Thouless which assumes that the transition is driven by a vortex–antivortex unbinding mechanism [5, 6]. Pairs vortices–antivortices are shown in Fig. 2. Another interpretation due to Patrascioiu and Seiler [14] came out later. They consider that the mechanism responsible for the transition is a polymerization of domain walls. Both interpretations were able to describe satisfactorily the observed transition. In this work, as a matter of unification of language and tradition, we use the terminology Berezinskii–Kosterlitz–Thouless transition and $T_{\rm BKT}$ for the transition temperature. A recent experiment [15] seemed to confirm the BKT picture in a trapped atomic gas where the authors

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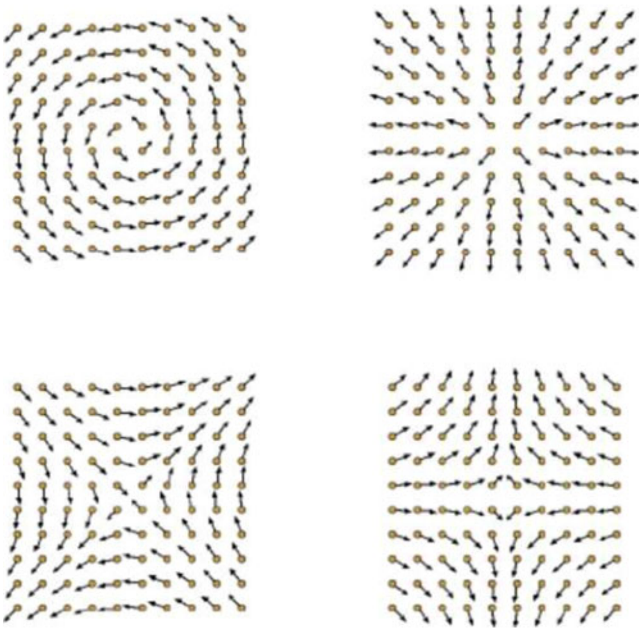

Fig. 1 Schematic view of ferromagnetic vortices of types I and II (*top*) and the respective antivortices (*bottom*) configurations. In an infinite system, all four configurations have the same energy

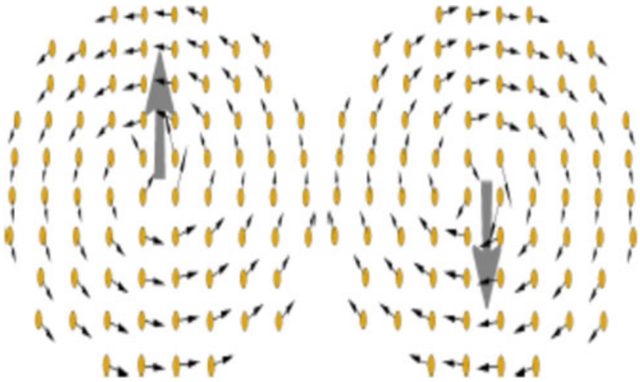

Fig. 3 Ferromagnetic vortices showing the out-of-plane component (*gray arrow*) and the polarization which can be positive (counterclockwise rotation) or negative (clockwise rotation). The direction of the out-of-plane component is independent of the polarization

attribute the transition to the proliferation of free local topological defects or vortices.

More recently, direct experimental evidence for the existence of magnetic vortex states in nano-disks was found by magnetic force microspin-polarized scanning tunneling microscopy and direct observation [16–21]. The interest in such models was renewed due to the possibility of applications in several fields, from spintronics to solar panels and biology. A quite special case is the use of magnetic nano-disks as bit elements in nano-scale memory devices [22–26]. In anisotropic ultrathin magnetic films, a few monolayers thick the ground state may have a vortex configuration. Depending on the anisotropy strength an out-of-plane magnetization develops at the center of the vortex, perpendicular to the plane of the disk, which can be "up" or "down" (see Fig. 3). Because the vortex structure is very stable, this twofold degeneracy make the nano-disk a promising candidate to build highly compact storage devices.

Although the properties of low-dimensional magnetic systems has experimentally been well studied and several models have been investigated by both theoretical and simulation means, there are still some properties that need a closer view, mainly those referring to their dynamical behavior. Our goal in this paper is to review some key points for the understanding of the vortex behavior and the progress that have been done in the study of vortices in low-dimensional magnetic systems.

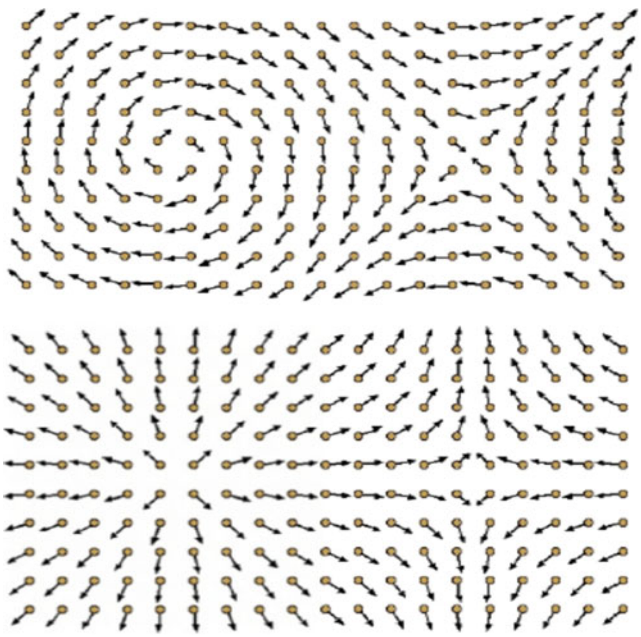

Fig. 2 Ferromagnetic pairs of vortex–antivortices of types I (*left*) and II (*right*). They are local excitations in contrast with the vortices that are global

2 The Vortex-Driven Phase Transition in the Anisotropic Heisenberg Model

The 2-D anisotropic Heisenberg model can be realized by several compounds as for example $BaCo_2(AsO_4)_2$ [27], stage-2 $CoCl_2$-GIC [28–32] and Rb_2CrCl_4 [33–35]. The simplest microscopic model capable of supporting vortex excitations is the classical two-dimensional anisotropic Heisenberg model described by the Hamiltonian [6]

$$H = \sum_{<i,j>} J_{i,j}\left(S_i^x S_j^x + S_i^y S_j^y + \lambda S_i^z S_j^z\right), \quad (1)$$

where, $\vec{S}_i$ is a classical three-component spin variable defined on the site i of a square lattice, $|\vec{S}_i| = 1$, J_{ij} is an exchange energy coupling spins at sites i and j and λ is an anisotropy. If $\lambda = 1$ we get the isotropic Heisenberg model that has no phase transition in two dimensions [12, 13]. For $\lambda > 1$, the model is in the Ising class-of-universality (easy axis); for $\lambda < 1$, it is in the planar-rotator class-of-universality (easy plane) undergoing a BKT phase transition. For $\lambda = 0$, in particular, we recover the so-called XY model, which should not be confused with the planar–rotator model, that has only two spin components. In the BKT picture [5, 6], this phase transition is believed to be driven by the binding–unbinding of pairs vortex–antivortex. In Fig. 1 we show a schematic view of ferromagnetic vortices and antivortices in a lattice. In Fig. 2 a vortex–antivortex pair is shown. A vortex is a global excitation with energy $\propto \ln R$, with R the vortex size, while a vortex–antivortex pair is a local excitation with energy given by $E_{v-av} \propto \ln R_{v-av}$, where R_{v-av} is the distance between the vortex and the antivortex centers. Because of that, in an infinite system, a vortex can only exist in the presence of an antivortex. In Fig. 4, we show the vortex density as a function of temperature for the two-dimensional anisotropic Heisenberg model. At low temperature (below T_{BKT}), vortices and antivortices form a condensate of pairs superimposed on a background of spin-wave excitations. At T_{BKT}, pairs shielded by the background start to *unbind* driven a transition to a *free* vortex phase. Above T_{BKT} the correlation length behaves as $\xi \propto \exp(bt^{-1/2})$ with $t \equiv (T - T_{\mathrm{BKT}})/T_{\mathrm{BKT}}$. Below T_{BKT}, $\xi \to \infty$ meaning that the model has a critical line at low temperature. The existence of the anisotropic term, $0 \le \lambda < 1$, does not change the behavior of the model. Both analytical as well numerical simulation results show that T_{BKT} depends weakly on λ, except for $\lambda \approx 1$ when $T_{\mathrm{BKT}} \to 0$. The critical behavior of this model is well discussed in the references [3, 5, 6]. The vortex structure developed in this model was first examined by Hikame and Tsuneto [36] and Homma and Takeno [37]. We follow those references to briefly reproduce here some important results that we will use in this paper. A continuum version of Hamiltonian (1) can be written as

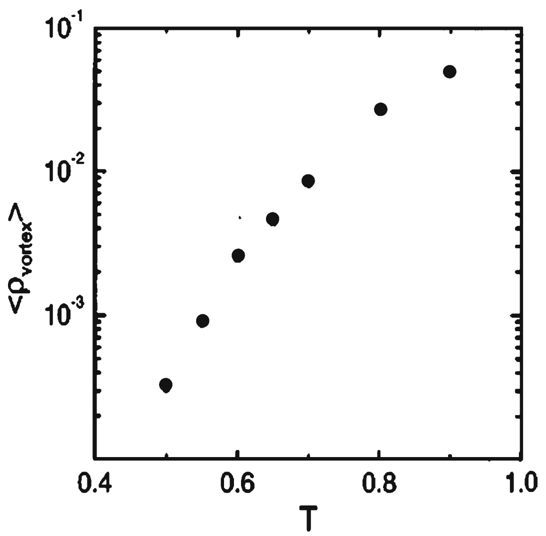

Fig. 4 Vortex density as a function of the temperature for the classical two-dimensional anisotropic Heisenberg model in a log-linear plot. (Extracted from [58])

$$H \approx \frac{J}{2}\int\left[\left(1 - \frac{\delta}{2}\cos^2\theta\right)(\nabla\theta)^2 + \sin^2\theta\,(\nabla\phi)^2 + \delta\cos^2\theta\right], \quad (2)$$

where $\delta = 2(1 - \lambda)$. The spin components were parameterized by using the spherical angles $\vec{S} = \{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$. By minimizing (2) in relation to the angles θ and ϕ one obtain

$$\nabla^2\phi = 0, \ (0 < \phi < 2\pi) \quad (3)$$

$$\left(1 - \frac{\delta}{2}\cos^2\theta\right)\left(\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr}\right) + \frac{\delta\sin 2\theta}{4}\left(\frac{d\theta}{dr}\right)^2 - \frac{\sin 2\theta}{2r^2} + \frac{\delta}{2}\sin 2\theta = 0. \quad (4)$$

It is easy to check that $\phi = \pm\arctan\frac{y}{x}$ is a solution for the first equation together with ferromagnetic boundary conditions. This kind of solution is named a vortex (+) or antivortex (−) (see Fig. 1). To get the out-of-plane component, θ, an analysis of (4) shows that $\theta = \pi/2$ satisfy the equation. Another solution can be obtained by inserting the vortex solution in (4) and assuming the boundary conditions: $\theta = \pi/2$ $(r \to \infty)$ and $\theta = 0$ $(r \to 0)$. Asymptotically, the solution for θ is

$$\theta \simeq \begin{cases} \frac{\pi}{2} - ae^{-\sqrt{\delta}r} & , \quad r \to \infty \\ b\,r^{(1-\frac{\delta}{2})^{-1/2}} & , \quad r \to 0 \end{cases}$$

A characteristic length scale is provided by $\frac{1}{\sqrt{\delta}}$ which can be interpreted as the vortex core. The energy of a single vortex can be estimated by using the solutions above. It reads

$$H \approx \frac{J}{2}\int_a^R \sin^2\theta\,(\nabla\phi)^2$$
$$\approx \pi J\left\{\ln\left(\frac{R}{a}\right) - \int_a^R \frac{\pi}{2}\frac{e^{-\sqrt{\delta}r}}{r}\mathrm{d}r\right\} \tag{5}$$

Here, a is an infrared cutoff, normally taken as the lattice size. If $\theta = \pi/2$ only the logarithmic term survives. The vortex energy diverges with diverging size. If $\lambda = 1$ ($\delta = 0$), the model turns out to the isotropic Heisenberg model and the vortex becomes an instanton with finite energy.

The discussion above shows the existence of two distinct types of vortices, one which has no out-of-plane spin component and another characterized by an out-of-plane spin magnetization at the core of the static vortex. The stability of these solutions were discussed by several authors [38–40]. It has been found that the in-plane vortex is stable for $\lambda < \lambda_c$, where λ_c is a critical anisotropy. Conversely, when $\lambda > \lambda_c$, the in-plane vortex becomes unstable, and develops into an out-of-plane vortex. It means that there is a region where the stable solution is $S^z = 0$ and another region where $S^z \neq 0$ near the vortex center. We observe that for $\lambda \approx \lambda_c$ the S^z component is noticeable only inside a small region near the vortex core. As long as λ increases, S^z becomes larger and the vortex core grows.

3 Vortex Dynamics in the 2-D Anisotropic Heisenberg Model

Although the static properties of vortices in the anisotropic Heisenberg model are well understood via several numerical and analytical works, the same is not true for its dynamical behavior. Most of the information about its properties are given by the space–time correlation function, $C(\vec{r}-\vec{r}\,', t)$, and its Fourier transform, $\mathcal{C}(\vec{q}, \omega)$ [41]

$$C^\alpha(\vec{r}-\vec{r}\,', t) = \langle S^\alpha_{\vec{r}}(t)\, S^\alpha_{\vec{r}\,'}(0)\rangle - \langle S^\alpha_{\vec{r}}(t)\rangle\langle S^\alpha_{\vec{r}\,'}(0)\rangle \quad , \tag{6}$$

and

$$\mathcal{C}^\alpha(\vec{q}, \omega) = \sum_{\vec{r},\vec{r}\,'} e^{i\vec{q}\cdot(\vec{r}-\vec{r}\,')}\int_{-\infty}^{+\infty} e^{i\omega t} C^\alpha(\vec{r}-\vec{r}\,', t)\frac{\mathrm{d}t}{2\pi} \quad . \tag{7}$$

Following the BKT picture, we expect that as temperature rises through T_{BKT} vortices start to unbind and diffuse, since vortices and antivortices are excitations superimposed on a background of spin waves. As a consequence, the pairs should diffuse through the Landau–Lifshitz equations of motion for each spin,

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{S}_i = \vec{S}_i \times \vec{H}_{\mathrm{eff}}, \tag{8}$$

with

$$\vec{H}_{\mathrm{eff}} \equiv -J\sum_j\left(S^x_j\hat{e}_x + S^y_j\hat{e}_y + \lambda S^z_j\hat{e}_z\right), \tag{9}$$

where the sum is over the first neighbors of $\vec{S}_i$ and $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are the unit vectors in the x, y and z directions respectively. As an alternative formulation the equations of motion can be obtained as the Euler–Lagrange equations of a functional [42–44]

$$K[\theta, \phi] = H[\theta, \phi] - \sum \dot{\phi}_i S^z_i, \tag{10}$$

where θ and ϕ are the spherical angles defined earlier. In the equation above the z component of the spins, S^z_i, is to be interpreted as the classical spin angular momentum. Because free vortices affect globally the system, the in-plane correlation functions, $\mathcal{C}^{xx} = \mathcal{C}^{yy}$, should be sensitive to the vortex–antivortex pair unbinding. On the other hand, as a consequence of ref. (10), if the separation cause the pairs to diffuse it is expected that they develop a z component to begin to move. Beside that, it is expected a stronger effect in $\mathcal{C}^{zz}$ when $\lambda > \lambda_c$, since the out-of-plane vortex becomes the most stable solution of the equations of motion as discussed above. The main results found in the literature so far are summarized in the following. For $\lambda = 0$ Villain [45] and Moussa and Villain [46] found the in-plane $\mathcal{C}^{xx} = \mathcal{C}^{yy}$ scattering function to have a δ function spin-wave peak at low temperature and a spin-wave peak $\mathcal{C}^{xx} \sim |\omega - \omega_q|^{1-\eta/2}$ close to T_{BKT}. Here, η is the critical exponent of the static spin–spin correlation function. Nelson and Fisher [47] treated the model without vortex contributions. They obtained a correlation function around the spin-wave peak as $\mathcal{C}^{xx} \sim \omega^{\eta-3}$. Menezes et al. [48] found a spin-wave peak similar to that obtained by Nelson and Fisher. In addition to the spin-wave peak they found a logarithmical diverging central peak, $\mathcal{C}^{xx} \sim 1/q\ln\omega$. This central peak was conjectured to be caused by vortex pairs diffusing on the lattice. Huber [49–51] discussed how a vortex gas approximation could contribute to a central peak to the Fourier transform of the spin–spin correlation functions in the hydrodynamic regime. Mertens [52] and co-workers calculated $\mathcal{C}(\vec{q}, \omega)$ above T_{BKT}, assuming an ideal diluted gas of unbounded vortices moving through the lattice. That phenomenology was successful in describing the central peak in one-dimensional soliton dynamics in magnetic spin chains

[7–9]. They found a squared Lorentzian central peak for $\mathcal{C}^{xx}$ and a gaussian central peak for $\mathcal{C}^{zz}$. Pereira and Costa [53] proposed a theory, based on pairs vortex–antivortex diffusion to describe the central peak below T_{BKT}. They found a Lorentzian central peak for $\mathcal{C}^{xx}$. They did not, however, consider the out-of-plane $\mathcal{C}^{zz}$ scattering function. Papanicolaou and Tomaras [54, 55] have calculated the dynamics of vortices deriving the equations of motion from the conservation laws of linear and angular momentum which are expressed as moments of a suitable topological density. As a result, a vortex is shown to be spontaneously pinned in the absence of external forces, while it would drift in a direction perpendicular to an applied uniform field at a speed calculable in terms of its initial configuration.

Several studies in neutron scattering on stage-2 CoCl2.Gic [28–32], have found strong evidences of a BKT phase transition. The wave vector and frequency were scanned carefully to obtain in details the spin–spin correlation function. Their results showed the expected central peak above the T_{BKT} temperature in the in-plane correlation function. The out-of-plane correlation function showed only spin-wave peaks. Evertz and Landau [56] carried out a very extensive and careful spin dynamics simulation on the 2-D−XY model. They found that the neutron-scattering function presented pronounced spin-wave peaks both in the in-plane and out-of-plane scattering functions over a wide range of temperatures. The in-plane scattering function also has a large number of clear but weak additional peaks which they interpreted as being from two-spin-wave process. In addition they observed a small central peak in the in-plane function at all temperatures, below and above T_{BKT}. No central peak was reported in $\mathcal{C}^{zz}$. Costa and Costa reported results of Monte Carlo and spin dynamics calculations [39, 40] for several values of λ. They found that there is a critical value of the anisotropy, $\lambda_c = 0.7035(5)$, above which appears a central peak in the out-of-plane, $\mathcal{C}^{zz}$, correlation function. This behavior was predicted for the first time by Hikame and Tsuneto [36]. Costa et al. have explicitly measured some microscopic vortex properties [57, 58]. They calculated the vortex density as a function of temperature and time, the vortex pair density as a function of distance between vortices and antivortices, the time interval for a pair to annihilate, and the time needed for a vortex to move one lattice spacing. Their results indicated that the vortices do not move extensively through the lattice, in close agreement with the results of Papanicolaou and Tomaras [54, 55]. Indeed, the vortices are subject to a strong creation-annihilation process moving only locally. Upon closer investigation they found that just before the vortices annihilate they develop a coherent out-of-plane spin component around the vortex core. They concluded that, if the vortices are really responsible for the central peak observed in the correlation functions, then these processes should play the defining role. Although there is a lot of results for the Heisenberg anisotropic model in two dimensions, the vortex contribution to the dynamics of the model is still an open question.

4 Dynamical Behavior of Vortices in Magnetic Nano-structures

As far as I know, the first report of a curly magnetic structure in a thin film was reported by E. E. Huber and collaborators [59] in 1958. They called the structure a “corkscrew”. Iwasaki and Takemura [60] suggested for the first time, in 1975, the possibility of using a circular mode of the magnetization for building high-density magnetic data recording. As was discussed by Papanicolaou [54, 55] and Compton [61, 62] an isolated vortex is naturally pinned in the lattice. However, when an external magnetic field is applied upon the vortex it can move in a direction perpendicular to the field (for an illustration see Fig. 3 For an oscillating field, it can rotate around its core equilibrium position at a characteristic frequency of several hundred megaHerz [63–65] (Fig. 5). A schematic view of this effect is shown

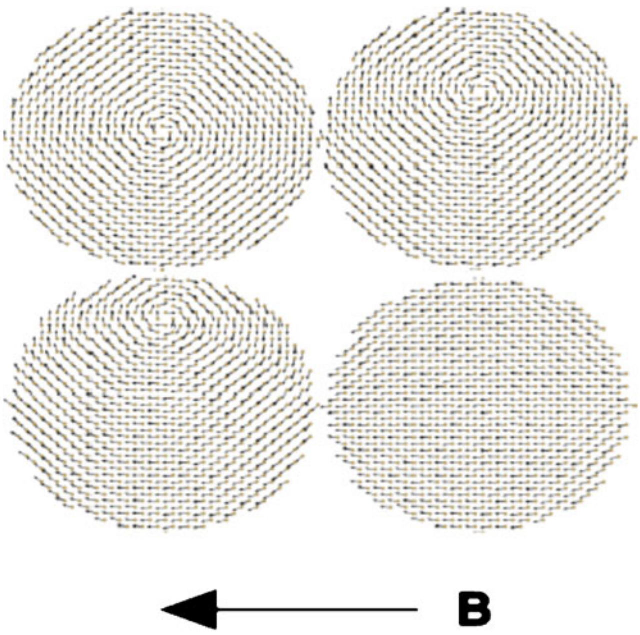

Fig. 5 Snapshots showing the behavior of a vortex subjected to the effect of an in-plane static magnetic field. The vortex is dislocated in the direction of $\vec{B} \times \vec{R}$ ($\vec{R}$ points out in the direction defined in Fig. 3)

in Fig. 6. A general Hamiltonian model for a magnetic nano-dot can be written in a pseudo-spin language as (for a revision on the subject, see [66–73])

$$H = \sum_{<i,j>} J_{i,j}\left(S_i^x S_j^x + S_i^y S_j^y + \lambda S_i^z S_j^z\right) + D\sum_{i\neq j}\left[\frac{\vec{S}_i\cdot\vec{S}_j}{r_{i,j}^3} - \frac{\left(\vec{S}_i\cdot\vec{r}_{i,j}\right)\left(\vec{S}_j\cdot\vec{r}_{i,j}\right)}{r_{i,j}^5}\right], \quad (11)$$

where $r_{i,j}$ is the distance between sites in the lattice and a dipole-dipole interaction with strength D has to be taken into account. For a finite system the continuity of the magnetic field in the boundary of the system imposes the magnetic moments to be tangent to the border of the nano-disk, so that, the ground state corresponding to Hamiltonian (11) has an impaired vortex at the center of the system. Much of the work done so far uses a variation of the Hamiltonian (11) by considering an anisotropic interaction $\sum(\vec{S}\cdot\vec{n})^2$ instead of the dipole term [74–76], with $\vec{n}$ representing a unit vector perpendicular to the surface and to the borderline of the system. It favors the magnetic moment into a configuration tangent to the border of the system and parallel to the disk plane. The energy due to this term is minimized when the magnetic moments arrange themselves in a curling vortex structure. The low-temperature properties of the hamiltonian with the anisotropic term is similar to the one obtained by using the long range dipole interaction. However, the high temperature and the dynamical behaviors are quite different. As discussed before, a S^z component can appears at the center of the vortex. It was found that there

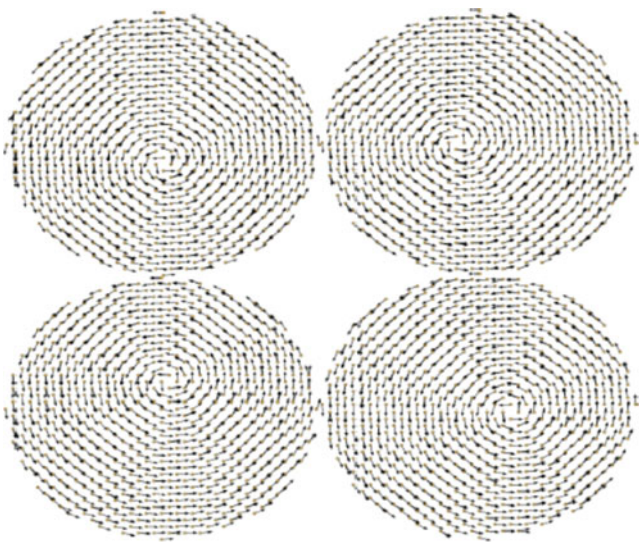

Fig. 6 Snapshot of the gyrotropic motion of a vortex

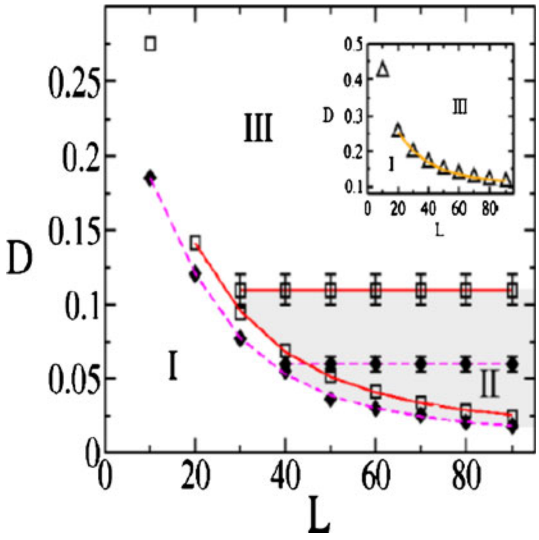

Fig. 7 Dipole interaction strength, D, as a function of the disk diameter, L, showing a diagram for the vortex formation for three different types of lattices. *Squares* and *diamonds* correspond to square and the hexagonal lattices, respectively. The *inset* are for a triangular lattice. Region I does not develop a vortex. Region III has a vortex in the ground state. The *shaded area* (region II) represents a region where the most stable configuration is a vortex with an out-of-plane component. Observe that this configurations does not exist for the triangular lattice

exist three regions where the most stable configuration can be ferromagnetic, an in-plane vortex or an out-of-plane vortex depending of the choice of the dipole strength [77] (see Fig. 7). Because the hamiltonian is invariant under a global operation $S^z \to -S^z$, the out-of-plane structure developed at the center of the vortex is degenerated and does not depend on the vortex orientation (clockwise or counterclockwise).

4.1 Gyrotropic Motion

A field-driven switching of the vortex polarization has been shown to be possible by applying a static magnetic field perpendicular to the vortex plane, thus pushing the vortex core and reestablishing it in the opposite direction [78, 79] (see Fig. 8). This process requires high field strengths of the order of 10^3 mT. Such a

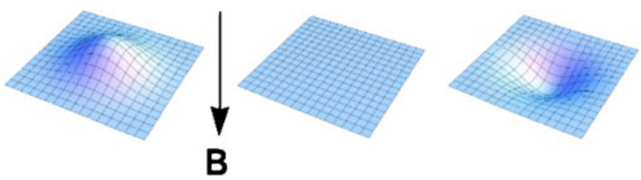

Fig. 8 A field-driven switching of the vortex polarization using a field perpendicular to the vortex plane. This process requires field strengths of the order of 500 mT

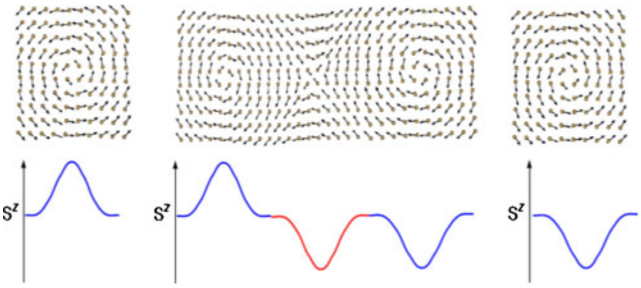

Fig. 9 Schematic of the process of pair-creation mediated vortex core inversion of the S^z component at the center of the vortex. A field pulse is applied in the disk plane. The vortex core magnetization is switched by a short magnetic field pulse applied in the film plane. This switching process requires only 40–50 ps

large field value indicates that a high energy barrier must be surmounted to switch a magnetic vortex core. This high barrier ensures high thermal stability, which in combination with their well-defined orientation and their extremely small size makes the vortex a strong candidate for binary data storage. The main problem to be surpassed is the effective control of the S^z component. More recently, a demonstration that vortex cores can also be switched by low-amplitude in-plane magnetic fields has been provided in an experiment of short oscillating magnetic field pulses of low amplitude tuned to the gyrotropic resonance frequency of the system [80–86]. The gyrotropic frequency depends on the particle size and shape and is typically of the order of 10^2 MHz. A weak oscillating resonant magnetic field induces a rotation of the vortex on a stationary orbit (see Fig. 6). Exploiting the sense of rotation, it was demonstrated that the vortex core reversal can be triggered with weak sinusoidal field pulses of about 4 ns duration and strength of about 1.5 mT [87].

Some numerical studies [87] suggested that the reversal mechanism is mediated by a process of creation and annihilation of a vortex–antivortex pair (see Fig. 9). A recent experiment using high-resolution time-resolved magnetic X-ray microscopy seems to give support to that mechanism [88].

5 Final Remarks

The study of models that can support vortex configurations reveals a very rich picture far of being completely understood. The richness of the models together with the possibility of technological applications invite us to look for a deeper comprehension of the phenomena.

Although we held ourselves in the study of ferromagnetic models, much of our discussion applies to anti-ferromagnetic models as well. On the other hand, fully frustrated models (FFM) lead to a still richer behavior [89–93]. The ground state of the two dimensional Heisenberg FFM is a condensate of vortices and antivortices. Beside the continuous spin symmetry it posses a Z_2 symmetry corresponding to the vortex degree of freedom which may lead to long range order.

The two-dimensional anisotropic Heisenberg antiferromagnetic model has a dynamical behavior similar to the ferromagnetic one. For the two-dimensional Heisenberg FFM only an exploratory study was done [94].

Finally, the dynamical behavior of a confined vortex, as in a nano-disk, has a strong appeal for technological applications as for instance, in the fabrication of non-volatile memory devices. Understanding the mechanism of the out-of-plane switching is of paramount importance for future applications.

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