The Microwave Background Radiation power spectrum

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Abstract

I discuss some aspects of the Microwave Background Radiation power spectra obtained by WMAP and Planck space observatories. Special attention is given to the anomalous power drop verified at the quadrupole anisotropy.

1 Introduction

Motivated by the publication of the partial results of the Planck space observatory [1], I turn now to a short discussion of the anisotropies of the Microwave Background Radiation (MBR), especially of its most popular representation, namely, its power spectrum.

The MBR can be very precisely represented by a 2.7 K blackbody. This was shown in an extraordinary way by the space observatory COBE (Cosmic Background Explorer), whose endeavor earned its team the 2006 Physics Nobel prize. The merit of such award is questionable, though, as I report in [2].

The MBR was discovered in 1964 by the American Arno Penzias and Robert Wilson, whom were awarded the 1978 Physics Nobel prize for this feat. Since then there was the suspicion that the radiation was of the thermal type, represented by the radiation spectrum of a blackbody, also known as the Planck spectrum. This was a homage to the German physicist Max Planck (1858-1947), that in 1900 discovered its theoretical formulation, the thermal radiation law. I discuss some aspects of the MBR in another article [3].

2 COBE

The observational advancements around the MBR occurred in a gradual fashion up to the launching of the COBE probe in 1989, aimed to its observation. The COBE observations represented the end of the issue, with the announcement of the results in the beginning of the 1990s. It turned definitive and extraordinarily demonstrated that the MBR is originated from a blackbody, which is very likely the most perfect blackbody in nature. The figure below illustrates this statement.

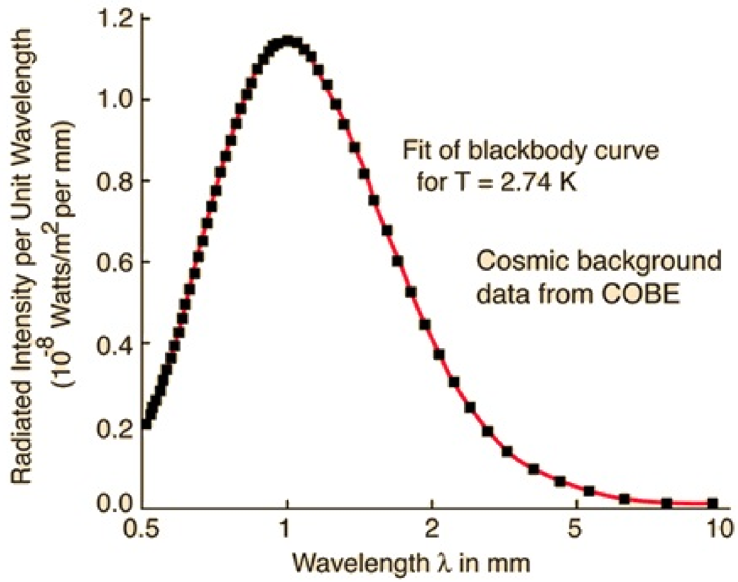

Figure 1: The Microwave Background Radiation observed by the COBE space observatory. The curve represents a 2.74 K blackbody, and the points with error bars are the experimental data. The fitting is the best experimental fitting of a blackbody ever obtained. It is always worth remembering that the fact that the MBR has a perfect blackbody spectrum does not imply that it is from cosmic origin as considered in the Standard Model of Cosmology (SMC). The original observation of Arno Penzias and Robert Wilson, the discoverers of the MBR, corresponds to a single point at $\lambda = 73.5$ mm, which was not among the COBE observations (COBE's team).

The COBE observatory also showed that the MBR was characterized by fluctuations of temperature all across the celestial sphere, the so-called MBR anisotropies. But the observations were of low resolution and sensitivity. In any case, the existence of the anisotropies was vigorously acclaimed by the SMC cosmologists, because they were extremely necessary in the cosmological scenario presented by the SMC. Such anisotropies are presumably indicatives of irregularities that existed in the distribution of the cosmic material when the MBR was formed, that is, when the universe was about 300,000 years old. It was from these primeval irregularities that the gravitational instabilities developed and ultimately gave origin to stars, galaxies and all other cosmic structures. Without them, we ourselves would not be here, according to the SMC followers. After the end of the COBE mission, a space observatory was proposed, the WMAP, to measure only the anisotropies, with larger resolution and sensitivity. With the great success of WMAP, new questions appeared, and larger resolution and sensitivity were required. Consequently the Planck satellite was launched and represents the current stage of our capacity of investigating the anisotropies. Surely enough new questions will emerge, which will demand larger resolution and sensitivity, and so forth. This is the proper character of scientific research.

3 WMAP and Planck

The MBR and its anisotropies are observed all over the celestial sphere. The MBR will be, for obvious convenience, identified by its average temperature and by the associated fluctuations. How to quantitatively represent them? There are several possible ways, but the most appropriate, due to the geometry of its spatial distribution over the celestial spherical surface, is the representation of the temperatures in a series of spherical harmonics. The series is characterized by the linear combination of multipoles identified by the multiplicity l (see below). The MBR intrinsic fluctuations will show up when the following components are removed from the observations:

- $\boldsymbol{l = 0}$ **:** ***monopole***− corresponds to the average value of the radiation. The average value is the temperature of a blackbody at 2.7 K, found by COBE.

- $\boldsymbol{l = 1}$ **:** ***dipole***− corresponds to the dipole anisotropy, which is caused, according to the SMC, by the motion of the Earth with respect to the microwave background.

Incidentally, the representation in a series of spherical harmonics is widely utilized in physics. It is used, for example, in the representation of the three-dimensional distribution of the electron cloud in a atom. The concept of an orbital appears from the geometrical representation of each component of the spherical harmonic series.

The anisotropy fluctuations are given by $\Delta T(\theta, \phi)/T_\circ$, where $T_\circ = 2.7$ K, $\Delta T(\theta, \phi) = T(\theta, \phi) - T_\circ$ and θ and ϕ are the spherical coordinates that identify the point of the celestial sphere for which the fluctuation is calculated. The fluctuation of the dipole component ($l = 1$) is of the order of 0.001 and the fluctuations of the higher order multipoles are minuscule,

$\Delta T/T_{\circ} \approx 0.00001$, corresponding to absolute fluctuations ΔT of the order of some tens of μK. The latter are the primeval, intrinsic, anisotropies associated to the temperature fluctuations existing at the time of the MBR formation.

The MBR power spectrum is a measure of the weight of each multipole in the spherical harmonic series expansion. To obtain it, one starts from the field of temperature $T(\theta, \phi)$, represented by the spherical harmonic series given by

$$T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \tag{1}$$

where (θ, ϕ) is any point on the celestial sphere and Y_{lm} is called *spherical harmonic function of degree l and order m*. The indexes l and m are associated to the spherical coordinates θ and ϕ, respectively. The power spectrum is defined as the mean squared value of the spherical harmonic coefficients:

$$C_l \equiv \langle |a_{lm}|^2 \rangle. \tag{2}$$

The addition theorem of the spherical harmonic functions and the fact that they constitute an orthogonal basis imply that the power spectrum will be independent of the azimuthal index m, being expressed only by the index l, which identify the multipoles of the anisotropies of temperatures.

In practice, the above power spectrum is calculated from the MBR observational data by calculating the products

$$\frac{\Delta T(\theta_1, \phi_1)}{T_{\circ}} \times \frac{\Delta T(\theta_2, \phi_2)}{T_{\circ}} \tag{3}$$

throughout all the celestial sphere. $\Delta T(\theta, \phi)$ is the temperature fluctuation at the point (θ, ϕ). The angular separation Θ between any two points, identified by pairs of (θ, ϕ), as in Eq. 3, is related to the multipole index of the spherical harmonic expansion in the following way:

$$\Theta \simeq \frac{180^{\circ}}{l}. \tag{4}$$

The maps of the anisotropies observed by COBE, WMAP and Planck are shown in Fig. 1 of the reference [4].

The figures below show the power spectrum of the MBR anisotropies observed by WMAP and by its successor Planck, whose results have already been published. The multipole corresponding to $l = 0$ — the monopole, i.e., the mean temperature of the field $T_\circ = 2.7$ K — and the dipole anisotropy at $l = 1$ are removed before the derivation of the power spectrum. The first point in the axis of abscissas corresponds to the quadrupole ($l = 2$).

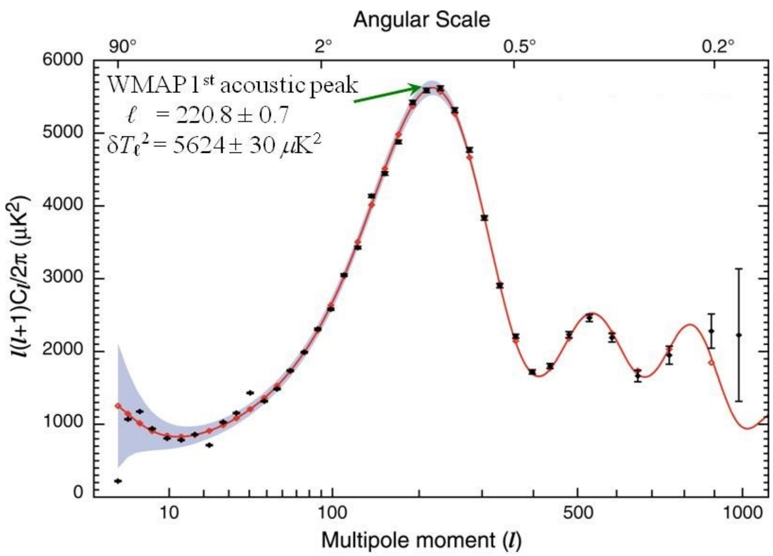

Figure 2: Power spectrum of the MBR anisotropies observed by WMAP. Notice the quadrupole power drop ($l = 2$), outside the SMC fitting, even considering the cosmic variance (see text), represented by the horn-shaped strip (WMAP's team).

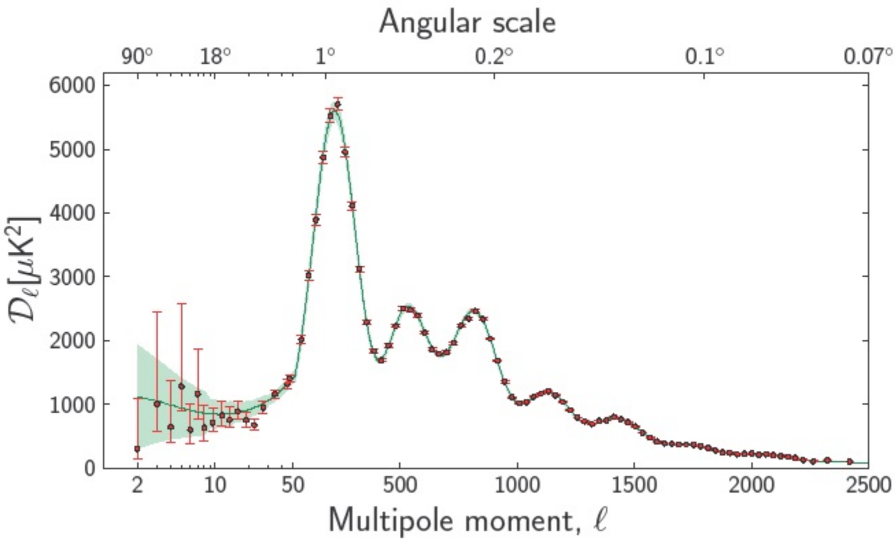

Figure 3: Power spectrum of the MBR anisotropies observed by Planck. The quadrupole power drop ($l = 2$) remains, being now marginally outside the SMC fitting, even considering cosmic variance (Planck's team).

What stands out on a first inspection is the increase in resolution and in sensitivity reached by Planck. The increase in the sensitivity is indicated by the increase of the largest multipole index, which are approximately 1,000 and 2,500 for WMAP and Planck, respectively. But first and foremost, a warning with respect to the apparent excellent theoretical fitting exhibited in both cases: the enormous number of free parameters of the SMC (almost 20!) allows for fitting essentially any set of data.

The spectra are characterized by a number of peaks, each one of them with a particular meaning in the SMC. Notice that all fluctuations described by the power spectrum are present in the observational data shown on Fig. 1, which are perfectly explained by a 2.7 K Planck spectrum. The fluctuations do not make any difference in the fitting, because the relative deviations introduced by them are, as we saw, of the order of 0.1% for the dipole anisotropy and of 0.001% for the other fluctuations. The dominant component in the data is the monopole ($l = 0$) and, as can be seen in Fig. 1, even with the presence of the fluctuations of higher multipoles the fitting by a blackbody spectrum is perfect.

The peaks of the MBR power spectrum, according to the SMC, represent the peaks of the acoustic oscillations — oscillations in a material medium —

of the primordial plasma. When the MBR was formed and set free from the plasma, these oscillations were impregnated in its energy distribution. The location of the first peak at $l \sim 200$ ($\Theta \sim 1°$), for example, is directly related to the total density parameter of the universe, that is, it informs us whether the universe is open, closed or critical. The results, both from WMAP and Planck, indicate that we live in a critical universe, which possesses, as we know, Euclidean global spatial geometry. The amplitude — not the location — of the second acoustic peak is directly related to the total density of baryons (essentially protons and neutrons) of the universe.

4 Anomalous quadrupole

Now, both WMAP and Planck present a serious problem, that was already explored in 2003 with the proposition of an alternative model to the SMC by the French cosmologist Jean-Pierre Luminet and collaborators. That is the *drop of power for the quadrupole anisotropy,* $l = 2, \Theta = 90°$ (eq. 4).

In some representations of the WMAP power spectrum that detail has been, in a certain way, hidden by the presentation form of the diagram — this is not the case, though, with the representations shown in Figs. 2 and 3. The renowned British theoretical physicist Roger Penrose, in his encyclopedic 2007 *"The Road to Reality"* [5], on page 775, in the legend of his figure 28.19, states *"Be sure to notice the very significant discrepancy at the quadrupole* ($l = 2$)*, almost hidden (accidentally?) by the vertical axis."* A reproduction of this figure is in Fig. 4. What is the grave implication of such a discrepancy that led the WMAP researcher team to the point of willing to hide it?

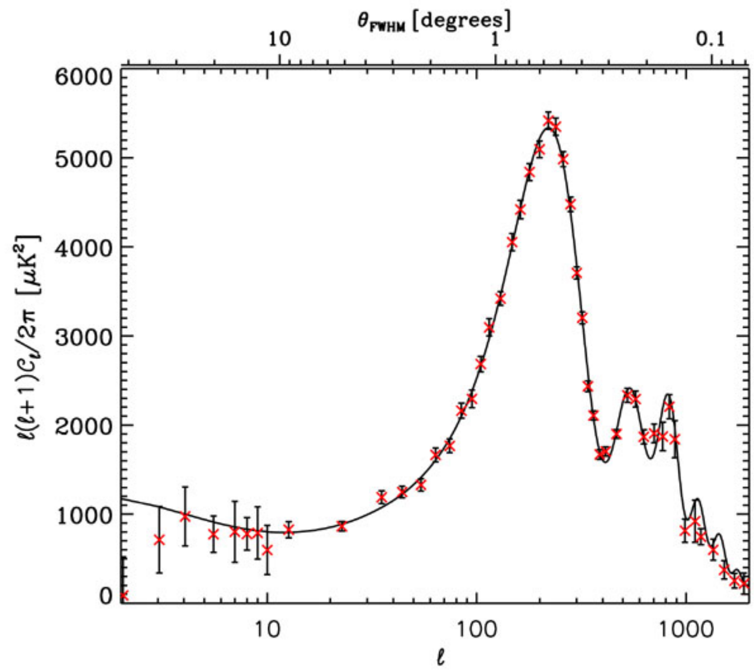

Figure 4: Figure 28.19 of reference [5] (see text). A symbol ×, close to zero, on the vertical axis, marks the quadrupole anisotropy observed by WMAP. Only an attentive eye may note this point, totally outside the theoretical fitting. The concept of cosmic variance was not yet used to justify the discrepancy, as in Figs. 2 and 3 (WMAP's team).

In general, the power drop for oscillations of large wavelength ($\geq 60°$) indicate that the universe might be finite. That means it has not sufficient size to support long oscillations. More or less like the impossibility of a violin producing the grave long waves of a cello. Note that, as mentioned above, the $l = 2$ quadrupole corresponds to an angular scale of the fluctuation $\Theta \approx 90°$. This goes frontally against the SMC that is spatially infinite and should exhibit the long oscillations with the same power. And what is worst, the observed discrepancy in the WMAP data was overwhelmingly confirmed by Planck. In 2003, immediately after the publication of the WMAP data, a French group, led by Jean-Pierre Luminet, caused uproar with the proposition of a relativistic cosmological model, alternative to the SMC, closed and with finite spatial section (see [6]). At the time, the model was called the "soccer ball model".

And how the SMC followers try to escape from this problem? Using a limitation that they assign to the physical characteristics of the problem, that is, a possible natural limitation called *cosmic variance*. As we saw, the problem arises repeatedly, in WMAP and in Planck. But what precisely is "cosmic variance"?

The cosmic variance is a natural uncertainty in the determination of the theoretical and observational power spectra due to the existence of a spatial limit of observation in the universe. The universe is limited by the finite size of the horizon of observable distance which, in the cosmological models of finite age, represents the maximum distance that we can observe from Earth. This results in that the statistical sampling of multipoles with small l (large angular scale) be small in comparison with the multipoles of small angular scale. The consequence of this is the introduction of an intrinsic statistical error that will be greater the lower the value of l. The horn shades, that appear in Figs. 2 and 3, represent these errors on the theoretical curve and, as one can see, it is maximum for the multipole component ($l = 2$).

There are at least two possibilities: either the observed discrepancy of the quadrupole, in the framework of the SMC, is assigned to the cosmic variance or it is assigned to the spatial finiteness of the universe — as Luminet and collaborators have done —, in which case the cosmic variance would have little influence. Of course, other alternatives to the SMC to explain the discrepancy certainly might exist.

In the case of the consideration of the cosmic variance in the SMC, we clearly witness an anomaly. The cosmic variance throws uncertainties upwards and downwards, but what is systematically seen in WMAP and in Planck is a downward difference of the quadrupole power. Hence, that would be an indication that there is not a problem of insufficient sampling, but instead the result of a spatially finite universe. Obviously, the SMC champions will never accept this, but will put forward, as an alternative, yet another launching of a spatial observatory, with larger resolution and sensitivity, in order to establish the question. Solving or not the question, this is very good for the science of cosmology, because better data soon would be available.

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