

# Object of Gravitational Extreme

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## Abstract

I present an alternative for the identification of “Schwarzschild’s singularity”, usually known as “black hole”, a name coined by the physicist John Archibald Wheeler. I propose to associate Schwarzschild’s singularity to the designation “object of gravitational extreme”. The fundamental difference between the object of gravitational extreme and the black hole is that the former has conventional space-time structure.

## 1 Introduction

Immediately after the conclusion of the General Relativity Theory (GRT) by Albert Einstein (1879-1955) in 1915, the German astronomer and physicist Karl Schwarzschild (1873-1916) derived a particular solution of GRT’s field equations that revealed itself to be extremely important. He had the intention to apply it to stars without rotation (or with negligible rotation) and perfectly spherical. His solution exhibits a *singularity* in the metric equation, i.e., an infinite result for a given value of the spatial coordinates. Generally speaking, the *metric* represents the space-time geometry of any space-time and gives the way of calculating the distance between any two events, which are characterized by three spatial coordinates and one tempo-

ral. Schwarzschild’s metric is given by (equation 6 of [1]):

$$(ds)^2 = -(1 - 2GM/rc^2)(cdt)^2 + \left(\frac{1}{1 - 2GM/rc^2}\right)(dr)^2 + (rd\theta)^2 + (r \sin \theta d\phi)^2, \quad (1)$$

$M$  is the body mass,  $G$  is the universal gravitational constant and  $c$  is the speed of light in vacuum. One sees immediately that Schwarzschild’s metric  $ds$  diverges to infinity at  $r = R_S = 2GM/c^2$ , where  $R_S$  is called *Schwarzschild radius* ([2, sec. 2]).

Einstein and the English astrophysicist Arthur Eddington (1882-1944), the greatest authorities in GRT in the early decades after its formulation, rejected any physical meaning associated to the singularity for obvious reasons (see more details in [3]). Researches related to the “Schwarzschild singularity” only began in the decade of 1960, especially from 1967 onwards, when John Archibald Wheeler (1911-2008) coined the term “black hole” (BH) to identify them.

The BH soon started to have status of “real object” and several physical, astronomical and cosmological consequences were deduced from it. But the BH is fundamentally, and will always be, the name of the “unknown”, since it is associated to a mathematical singularity. Such an object, therefore and indeed, does not exist. If BHs do not exist, so what? What astronomical object might be associated to the Schwarzschild singularity? That is what I intend to answer.

In the next section I present some features of the SS that are usually assigned to BHs. Section 3 is dedicated to the answer of the question put forward here. I finish with some additional remarks.

## 2 Schwarzschild’s singularity

The SS is currently identified with the BH. Normally one says that a BH is an object of extremely high density. It is important to clarify such an idea before we find a better physical conception for the SS, as proposed in the previous section. We shall see that an SS — the BH — can have extremely low density.

Let  $\rho_S$  be the mean density inside a sphere of radius equal to the Schwarzschild radius, that is,  $\rho_S = M/[(4/3)\pi R_S^3]$ . Consider now BHs that are often discussed in modern scientific literature:

- 1) BH of stellar mass  $M_S = 1 M_{Sun}$  and  $R_S = 3$  km,
- 2) BH of mass equal to the BH believed to exist in the center of the Milky Way with  $M_{MW} = 4 \times 10^6 M_{Sun}$  (4 million solar masses) and  $R_S = 1 \times 10^7$  (10 million) km and
- 3) BH of mass equal to the BH believed to exist in the center of the giant elliptical galaxy M87 with  $M_{M87} = 6 \times 10^9 M_{Sun}$  (6 billion solar masses) and  $R_S = 2 \times 10^{10}$  (20 billion) km.

(Schwarzschild radii  $R_S$  were calculated with the expression  $R_S = 3 (M/M_{Sun})$  km, cf. [4, sec. 2.2].)

Figure 1 shows the location of these BHs on the diagram  $\rho_S \times R_S$  ( $\rho_S \propto 1/R_S^2$ ; the scales of the axes are expressed as logarithms of the coordinates). The density of water ( $H_2O$ ) is marked and one can see that we can have BHs with any mean densities, much larger and much smaller than the density of water. In particular the putative BH of stellar mass has  $\rho = 2 \times 10^{16}$  (20 thousand trillion)  $g/cm^3$ , the BH at the center of the Milky Way has  $\rho = 10,000$   $g/cm^3$  and the BH at the center of M87 has density equal to  $0.0004$   $g/cm^3$ .

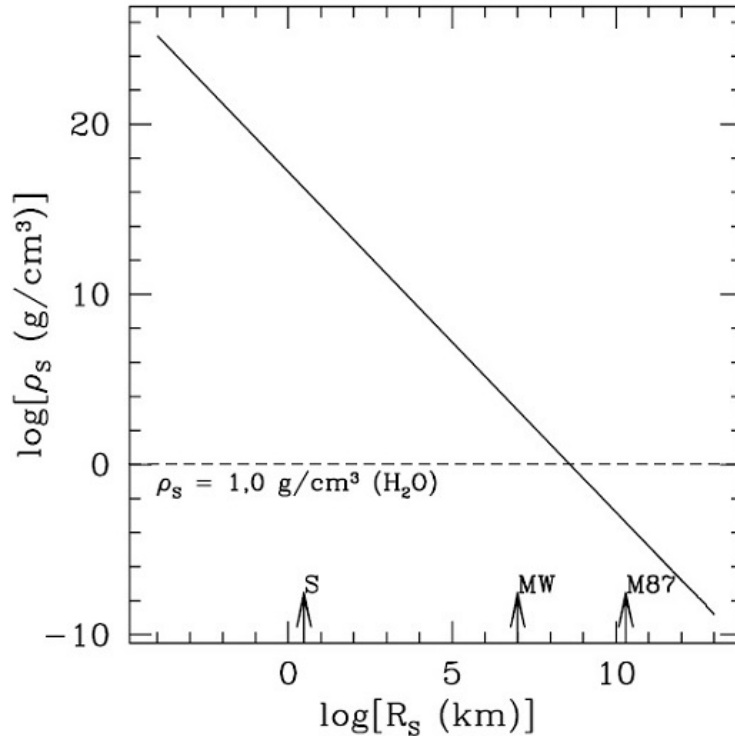


Figure 1: Schwarzschild's singularities in the diagram  $\rho_S \times R_S$ : S, mass equal to  $1 M_{Sun}$ ,  $R_S = 3$  km, MW, mass equal to  $4 \times 10^6 M_{Sun}$ ,  $R_S = 1 \times 10^7$  km  $\approx 15 \times R_{Sun}$  and M87, mass equal to  $6 \times 10^9 M_{Sun}$ ,  $R_S = 2 \times 10^{10}$  km  $\approx 130 \times d_{Sun-Earth}$ .

This analysis shows that the object associated to the SS does not necessarily have extremely high density. The object cannot be also associated to  $r = R_S$  because we have there an infinite value for the metric. Hence the object must be defined for  $r > R_S$ , but as close as to  $R_S$  as one wishes, i.e., the object is to be defined for  $r \rightarrow R_S$ . Let us go now to its definition.

### 3 Object of Gravitational Extreme

In the neighborhood of  $r = R_S$  the space-time curvature is very large. The space-time curvature is GRT's representation of the Newtonian gravitational

potential. In fact, one has for  $r \rightarrow R_S$  an extreme space-time curvature and, therefore, a *gravitational extreme*. Or else, let us see.

If we are so close to the SS as we wish we have, thus,  $r \rightarrow 2GM/c^2$ , whose terms may be rearranged in the form  $GM/r \rightarrow (1/2)c^2$ . We see then that  $\mathbf{M}/\mathbf{r}$  ( $\propto$  gravitational potential) tends towards an **extreme** ( $c^2/2G$ ). Whereas  $M/r^3$  ( $\propto$  density inside the sphere of radius  $r$ ) may be, as we saw, **large or small**. In other words, the packing of matter-energy is an extreme, but that does not necessarily mean that the density is an extreme inside  $r = R_S$  as well. What is an extreme is the packing and consequently the gravitation in the neighborhood of  $r = R_S$ .

From what has been said above the new object, that exists immediately before Schwarzschild's radius, may be appropriately called "object of gravitational extreme" (hereafter "*gravex*"). Before achieving the stage of a gravex we can have objects with large gravitation like white-dwarf stars and neutron stars, if we consider only objects of stellar masses. These stars are compact ones but have radii that are larger than the Schwarzschild radii corresponding to their masses.

Figure 2 illustrates the space-time configuration of the gravex next to white dwarfs and neutron stars. The Schwarzschild radius defines, in this new context, a spherical surface called "horizon of gravitational extreme"; a gravex, of any mass, by definition, never reaches such horizon. Gravex, like BHs, may have stellar masses, be microscopic or supermassive.

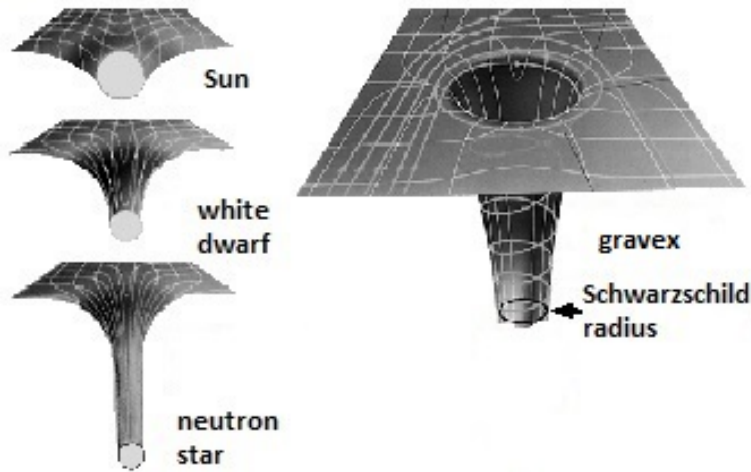


Figure 2: GRT shows us that space — more precisely, space-time — curves itself near any body. It is as space were a rubber sheet and the bodies “sank” on it. Above we see space curvature around the Sun, a white-dwarf star, a neutron star and a gravex. Notice that the gravex is formed before the “horizon of gravitational extreme”, defined by the sphere whose radius is “Schwarzschild radius” shown in the figure (adapted from figure 3 of [4]).

As we see in figure 2, the gravex have conventional space-time structures similar to those of white dwarfs and neutron stars. The stellar-mass gravex may be called *stellar extremes*, or simply *extremes*, and represent the next objects in the sequence of stellar objects of large gravitation already known. The BHs are the ones that occupy this place in the vision of orthodox modern science. As mentioned above, one can talk also of microgravex and super-massive gravex.

There are two important questions to be addressed with respect to the gravex.

- 1) What is the physical mechanism (the force field) that support the structure of a gravex? We know the mechanisms for white dwarfs (degenerate electron repulsion) and for neutron stars (degenerate neutron repulsion and repulsion due to the strong interaction, the same that exists in the atomic nucleus).

The answer to the first question pave the way for the formulation of the

second.

- 2) Will be a gravex stable or will not be the case that, from a given limit mass, a gravex breaks itself in stable gravex of smaller masses?

These questions are intellectually more satisfactory than living with the unknown scenario in which the BH inhabits, always waiting for the advent of a redeeming new theory of gravitation that solves the problem of the singularity. The gravex avoids the singularity and may — or not — be the definitive answer to the question of objects with large gravitation.

## 4 Final remarks

The existence of the SS denounces in a clear way the necessity of a quantum gravity theory (QGT) and emphasizes the precariousness of the classical theories of gravity such as GRT and Newtonian gravity (see an ampler discussion about this aspect in [4]). A QGT would expand the scope of the Schwarzschild solution in the domains constrained by the existence of the singularity.

The mean densities of the SSs illustrated in figure 1 cover an enormous range, from  $10^{-4}$  g/cm<sup>3</sup> (supermassive SS) to  $10^{16}$  g/cm<sup>3</sup> (stellar SS). In this range we have, in absolute terms, very small values and very large values. Nevertheless, it is worthwhile pointing out that such values are extraordinarily large when compared to the mean densities of the sites where it is assumed that the SSs are found. For example, the mean densities in galaxies range from  $\sim 10^{-24}$  g/cm<sup>3</sup> in the outer regions to  $\sim 10^{-21}$  g/cm<sup>3</sup> in the nuclear regions. A stellar SS located in the external regions has a mean density of about  $10^{40}$  times as large as the mean density of its neighborhood, whereas a supermassive SS located in the center of a galaxy has density about  $10^{17}$  times as large as the galactic nuclear density. This is the reason why SSs are usually associated to objects of enormous densities, that is, extremely compact, instead of to objects of gravitational extremes, as it is conceptually more appropriate.

Supermassive gravex resemble the “superstars” discussed by the American physicist Richard Feynman (1918-1988) in his book *Lectures on Gravitation* [5], especially in Lecture 14. Feynman’s Lectures were given in the beginning of the decade of 1960, well before the BH “fever” being installed.

Feynman has verified that the superstars are unstable relativistic objects. Incidentally, it is worthwhile reading the comments by physicists John Preskill and Kip Thorne presented in the *Lectures* preface. This preface is by its own an interesting lecture on gravitation and is available in [6].

As Einstein in 1939, Feynman was concerned about the SS issue. On p. 156, section 11.4, he states the problem: “*The metric eq.(11.3.6) [eq. 1 here] has a singularity at  $r=2m$  [ $r = 2Gm/c^2$  in my system of unities]. To find out whether this is a physically troublesome or meaningful singularity, we must see whether this corresponds to a physical value of the measured radius from the origin of the coordinates (which is not the same as our coordinate  $r!$ ) (...)*” The result of his analysis is the permanence of the singularity, now in another system of coordinates. What is impressive to me in Feynman words above are the expression “*physically troublesome or meaningful singularity*”. A “*physically troublesome*” singularity must be discarded, this is easy to understand. But what is a “*physically meaningful singularity*”? Can, to begin with, a singularity be “*physically meaningful*”? Feynman’s calculations — and common sense — seem to show it cannot. Again, one sees here the need of a QGT to solve the problem and eliminate the singularity.

In section 11.5 of Feynman’s Lectures we have a curiosity in the realm of “science digression” (cf. [4]). Feynman presents an initial discussion about the possible extrapolations around SSs, which at that time were, according to him, “*called ‘wormholes’ by J.A. Wheeler*”. J.A. Wheeler is the already-cited John Archibald Wheeler, whom some years later would christen such “possible extrapolation” as “black hole”, giving up the initial denomination of wormhole. This, afterwards, would be applied to another extremely aberrant object (see [4, sec. 2.1]). It is worthwhile remembering that the Lectures were given in the years 1962 and 1963 and that Wheeler invented the black hole in 1967, as history records.

**Acknowledgment** — Jos Victor Neto, subscriber and frequent commentator of my cosmology list **COSMOS** (<http://www.fisica.ufmg.br/dsoares/cosmos/cosmos.htm>, in Portuguese), brought to my knowledge a most interesting book by Feynman *Lectures on Gravitation*, which I mention in this article; I certainly share his enthusiasm for this remarkable work.



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