The MBR and the blackbody spectrum

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July 20, 2010

Abstract

I discuss some aspects related to the Microwave Background Radiation (MBR), in the framework of the standard model of cosmology, also called the *Big Bang model*.

The universe, according to the standard model, evolves from a phase dominated by radiation to a phase dominated, first, by matter and, presently, by dark energy.

After entering the matter age, another great transition occurs. The radiation that until then was strongly coupled to matter suffers a progressive transition to the total decoupling and starts to travel completely unimpeded through the expanding universe.

An extraordinary fact: the thermal equilibrium during the coupling phase generated a distribution of radiation according to a Planck spectrum — the blackbody spectrum. After decoupling the radiation dilutes itself, but preserves these characteristic spectrum until present days.

The first time that the thermal spectrum occurred was in the cosmological photosphere, when the decoupling was in full development. This is characterized by a transition from an insurmountable barrier due to Thomson scattering, prevailing until the phase of total matter decoupling, when the radiation began to propagate completely free.

Several questions may be posed. Some:

1- The blackbody spectrum is generated in the phase of matter-radiation coupling. How its preservation, in the following phase of expansion and free propagation of the radiation, is demonstrated?

During the expansion, the wavelength of the radiation λ increases proportionately with the increase of the scale factor $a(t), \lambda \propto a(t)$. A whole number of wavelengths must fill the universe at every instant, as illustrated in the figure.

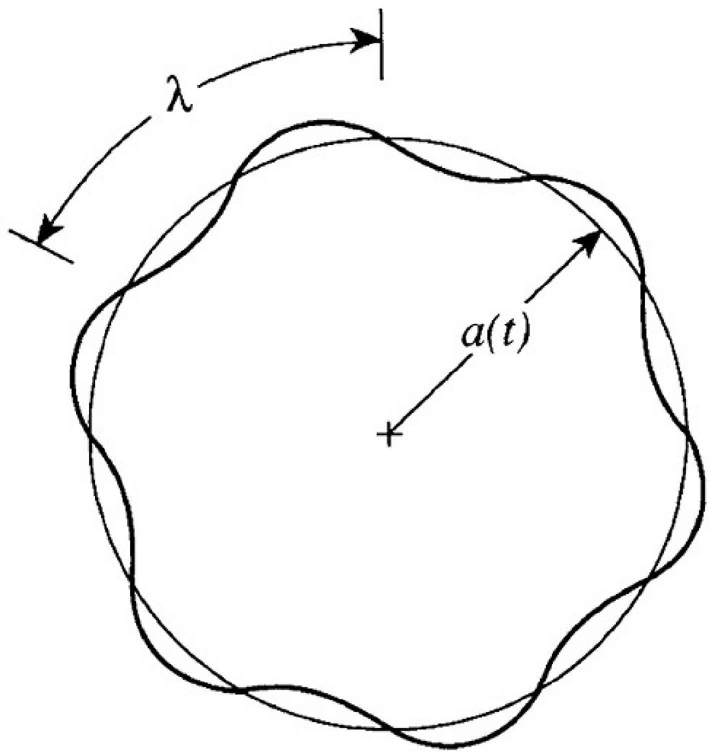

Figure 1: Unidimensional model for the oscillation of a wave. The closed spatial section of the universe is represented by a circle of radius equal to the scale factor of the universe $a(t)$. The wavy line represents the spatial part of an electromagnetic wave. The wavelength of this oscillation mode is proportional to the scale factor, that is, $\lambda \propto a(t)$ (cf. figura 5.8 of [1, p. 97]).

The astrophysicist P.J.E. Peebles [1, p. 134] starts from this and presents two explanations for the preservation of the MBR blackbody spectrum after decoupling, and during the expansion. The most simple of them is the following.

In a given temperature T, the occupation number, or mean number N of photons per oscillation mode (frequency), is given by the Planck function,

$$N = \frac{1}{e^{h\nu/kT} - 1} = \frac{1}{e^{hc/kT\lambda} - 1}. \tag{1}$$

In the range of observed wavelengths, the MBR frequencies are much greater than the universe rate of expansion, given by the Hubble parameter H. This means that the time evolution of the MBR wavelengths is adiabatic, and then, in the absence of interactions with other fields, the number of photons in each oscillation mode is conserved, i.e., N is time-independent.

Hence, being N constant, one has $T\lambda =$ constant (cf. eq. 1), and the temperature associated with the mode must obey the scale relation

$$T \propto \frac{1}{a(t)}, \tag{2}$$

because, as we saw previously, $\lambda \propto a(t)$.

During the expansion, the temperature of the radiation is, therefore, independent of the wavelength. Since the radiation is initially in thermal equilibrium, the temperature is the same for all modes and stays independent of the wavelength. This means that the radiation spectrum remains a blackbody spectrum, as initially.

Finally, one concludes that the MBR temperature has the following dependence with the expansion of the universe, given by the scale factor $a(t)$:

$$T(t) = T_\circ \frac{a(t_\circ)}{a(t)} = T_\circ(1+z), \tag{3}$$

where z is the redshift, given by $1 + z = a(t_\circ)/a(t)$.

The matter-radiation decoupling occurred at the temperature of ≈ 3000 K [2, cap. 29]. The current temperature ($z = 0$) is 3 K, thus the decoupling happened at the redshift of approximately 1000. This is the larger redshift which is possible to be accessed by means of electromagnetic waves, according to the Big Bang model.

The observational determination of the MBR spectrum is a task of extreme experimental complexity. The satellite COBE realized this task in a brilliant way, but there has not been yet an experimental repetition of the feat. The WMAP and Planck missions are dedicated to the measurement of the MBR anisotropies.

> **2- Why would the first structures have formed around the decoupling phase? (e.g., [3, cap. 6]). After all, the condition of predominance of gravitational pressure over radiation pressure is a function of the matter density and, in principle, may occur at any time.**

No, it may not. The decoupling epoch has two features that contribute to favor the gravitational collapse, namely, (i) the radiation exerts less pressure and (ii) the matter can collapse and irradiate the gravitational energy lost in the collapse via electronic transitions, because matter is not fully ionized anymore. Before the decoupling phase, the collapse would be interrupted due to the enormous thermal energy generated by the loss of gravitational energy, and that could not be irradiated, since the matter was fully ionized. The energy conservation during the collapse must be obeyed, which implies on the deposition of the lost gravitational energy in kinetic ("thermal") energy of the matter in the process of gravitational collapse. Besides, we have the tremendous radiation pressure, because the radiation is strongly coupled to the matter.

References

[1] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, 1993)

[2] B.W. Carroll, D.A. Ostlie, *An Introduction to Modern Astrophysics* (Addison-Wesley Publ. Co., Inc., Reading, 1996)

[3] R.E. de Souza, *Introdução à Cosmologia* (EDUSP, São Paulo, 2004) (in Portuguese).