A stone of stumbling to General Relativity Theory

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Abstract

The Standard Model of Cosmology represents a serious challenge to General Relativity Theory. Despite its great success in other applications, Einstein's theory apparently fails in this model. From a number of observational evidences, with respect to the energy budget of the universe, it is fair to say that the Standard Model of Cosmology does not deserve the status it has amongst a large fraction of the scientific community. I discuss the characteristics of General Relativity Theory that are responsible for its overwhelming success as a physical theory and those that point to its failure. The overall discussion is focused in two special solutions of General Relativity, which may be illustrated by the spacetime metrics associated to them: Schwarzschild's metric and Friedmann-Robertson-Walker's metric.

1 Introduction

General Relativity Theory (GRT) is a theory of great success due to the support of innumerable experimental evidences.

Such a statement has a subtle bias, though, that many times leads to the neglect of a particular failure of GRT, namely, its cosmological application. It is in this context that it is reasonable to say that cosmology, *the science* *of the universe*, is truly a stumbling block to GRT. That is what I intend to show in what follows.

The two most known — and, presumably, fundamental — applications of GRT are described by two metrics: Schwarzschild's (SM) and Friedmann-Robertson-Walker's (FRWM). The first of them is responsible for the large triumph of the theory. I present in the following section some aspects of SM. Next, I discuss FRWM and the reasons that make it an application of GRT that does not work in the real world, in other words, that does not comply with the observables derived from nature.

2 Schwarzschild's metric

In 1916, soon after the publication of Einstein's papers on GRT, the German astronomer Karl Schwarzschild (1873-1916) solved Einstein's field equations for a very special case, at the same time simple and of great applicability both experimentally and observationally. It refers to the determination of the spacetime metric in the exterior of a static and spherically symmetric mass distribution M. One should notice that Schwarzschild's solution is a vacuum solution, outside the object with mass M, and valid only in this region of spacetime. It is described in a simplified way by the expression of the spacetime interval ds:

$$(ds)^2 = \left(cdt\sqrt{1 - 2GM/rc^2}\right)^2 - \left(\frac{dr}{\sqrt{1 - 2GM/rc^2}}\right)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2,$$

where r, θ and ϕ are the usual spherical coordinates.

This metric is successful in its applications. It is verified in planetary motion, in the deflection of light due to presence of a mass concentration, in the correct prediction of the advance of Mercury's perihelion — where Newtonian gravity breaks down — and in modern applications of global positioning systems (GPS). Its singularity at the so-called "Schwarzschild radius", $r_S = 2GM/c^2$, raises theoretical discussions on a plausible inhabitant of the natural world, that is, the well-known "black hole".

Summing it all up, SM is the major responsible for GRT's enormous prestige in the scientific community, specially amongst the physicists.

But there is another GRT's application where difficulties are everywhere increasing. The metric within a homogeneous and isotropic fluid has an obvious application: cosmology. It is there that GRT finds, unfavorably, its first great experimental confront.

3 Friedmann-Robertson-Walker's metric

The mathematical complexities of GRT's application to cosmology are largely attenuated by the adoption of the *Cosmological Principle*.

The Cosmological Principle states, in simple terms, that all matter in the universe is uniformly and equally distributed in every spatial direction. In building cosmological models one neglects the fact that matter sits in stars, that stars are grouped in galaxies, and so forth, in larger and larger structures. Everything is "softened" and "smoothed out", by the invention of the "cosmic fluid", which is distributed in a homogeneous and isotropic manner throughout the universe. Its matter and energy density coincides with the real average density of stars, galaxies and clusters in the whole cosmos.

Spacetime is curved outside a mass concentration, as seen in the Schwarzschild metric. Likewise, spacetime is curved in the inside of a continuous mass distribution. The Standard Model of Cosmology (SMC) uses such a simplified perspective in the application of GRT to the universe.

One of the basilar experimental facts for the SMC is the discovery by Edwin Hubble that spectra of distant cosmic objects are systematically shifted towards the red extreme of the electromagnetic spectrum. The larger the shift, the more distant is the object — or, as stated in the original discovery, the fainter is the object. Such a phenomenological behavior became known in the scientific — and popular — literature as *Hubble's law*.

In the formulation of the SMC, Hubble's law may be interpreted in various different ways, due to the freedom of choice of the cosmological coordinate system. It is instructive to read what Bertrand Russell wrote in "The ABC of Relativity" ([1], chap. XI, "The Expanding Universe", p. 105) on this particular issue:

> "Einstein's law of gravitation, combined with the smoothing-out assumption — the assumption of exact uniformity — allows us to construct a variety of models of the universe, in which the overall curvature takes a variety of different forms. The main effect of this overall curvature is that it implies, in some of the models, that the universe will look as if it is expanding. There is a certain freedom of choice

> here, because of the freedom of choice of co-ordinate systems which we have in relativity theory. We may choose the co-ordinates so that the smoothed-out matter is at rest, and the space-time has a certain curvature, or we may choose them so that the matter is expanding, and the curvature appears to be reduced. It is a matter of taste which sort of co-ordinates we use, and it makes no difference to the end result, which is the prediction that according to these models of the universe, the spectra of distant objects will be observed to be shifted towards the red. We may attribute this red-shift to the expansion, or to the curvature, or partly to each. Because expansion is an easier notion to think of than space-time curvature, it is usually more convenient to speak of the expanding universe than the curved universe, but in mathematical terms the two concepts are the same. In the case of the predicted red-shift in the spectral lines of the sun, which was the effect we were concerned with in Chapter IX, it was more convenient to think in the other way, and attribute the red-shift to curvature."

The metric that describes spacetime in the interior of a homogeneous and isotropic fluid was derived by the Russian Alexander Friedmann (1888-1925), in the beginning of the years 1920s, when — like Schwarzschild did for the exterior of a spherically symmetric mass — he solved Einstein's field equations of the GRT, under the simplified assumptions of the Cosmological Principle. Later on, in 1935, the North American mathematical-physicist Howard Robertson (1894-1975) and almost simultaneously, the English mathematician Arthur Walker (1909-2001) showed that Friedmann's metric could be alternatively derived from purely geometrical arguments, as long as the fluid follows the Cosmological Principle. Hence, one has the so-called *Friedmann-Robertson-Walker metric* (FRWM):

$$(ds)^2 = (cdt)^2 - R^2(t)\left[\left(\frac{dr}{\sqrt{1-kr^2}}\right)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2\right],$$

where $R(t)$ is the scale factor and k the space curvature constant. The metric describes the typical expanding and curved models of the SMC. Of course, the metric admits also the possibility of stationary and contracting models.

The phenomenology of the radiation of distant objects, according to Hubble's law, is entirely consistent with the prescriptions of the FRWM, as discussed above by Bertrand Russell, being due to either a curvature or an expansion of the universe.

But there is a third — yet radical — possible interpretation for the phenomenon. It has not a definitive formulation, in terms of physical processes, but can be identified, in general terms, as a new paradigm, namely, the "tired-light paradigm". In this view, which is adopted in some alternative models to the SMC, light looses energy as it moves through the enormous cosmic distances since it is emitted from remote sources. How this happens, or, what is the physical process working is still a matter of debate. Incidentally, this was the idea that Hubble himself believed, when the first relativistic cosmological models were put forward (see a thorough discussion on this in [2]). The tired-light paradigm should be seen in such a general way and not related to particular physical process that justifies it. Alternative theories to SMC may adopt different physical processes, within the tired-light paradigm, in order to account of Hubble's law.

Currently, it seems, however, that there is no need whatsoever of an alternative theory to the SMC. The FRWM is considered as physically plausible — and esthetically satisfactory — to the explanation of a variety of phenomena observed in the universe, including Hubble's law. Thus, where is the stumbling stone? How does it appear in the path of GRT's FRWM?

4 The stumbling stone

There are many stones. In fact, GRT's path followed by the hands of the SMC is a torrential river of stones and monumental blocks. I shall describe two of these stones, which are truly solid rocks and, indeed, of arduous — but not impossible, in principle — remotion.

The SMC makes a well-defined prediction with respect to the matter and energy contents of the universe. The model is supported by the following matter-energy budget (see [3], for details):

- Bright stars (observed baryonic matter): 0,5%
- Baryonic dark matter: 3,5%
- Nonbaryonic dark matter: 26%
- Dark energy (non-electromagnetic energy): 70%

The figures listed above vary slightly from author to author but the substance is the same. What calls one's attention in the SMC budget is the predominance of matter and energy components that are not directly detected from observations or by means of experiments. They are those components called "dark". Out of three dark components, two are, additionally, entirely unknown: nonbaryonic dark matter and dark energy. They are discussed in the theoretical domain but there is no consensus of their real physical nature.

It is fair to say that the SMC rests upon the unknown, the first stone.

The second is known as "the problem of the cosmological constant". Dark energy is a radiation component that has a property which is diametrically opposed to its analogous in the familiar electromagnetic radiation. The latter goes into the energy balance of Einstein's field equations as an attractive contribution, from the gravitational point of view. A universe filled in only with electromagnetic radiation would have a positive curvature. Dark energy, on the other hand, has a repulsive effect on the spacetime, generating a negative curvature. There are a number of candidates — quantum fields — to be responsible for the dark energy. But the preferred option amongst the theorists of the SMC is the so-called *cosmological constant.*

Its story began with Einstein, who, in the first ever application of GRT to the universe, has introduced it in his field equations (see brief account of Einstein's model in [4]). His aim was to balance the attractive effect of matter and electromagnetic energy to obtain a static universe, which was his intuitive idea about the cosmos around 1917. Hubble had not yet made the great discoveries on the existence of galaxies and the law of redshifts.

Nowadays, the SMC does not advocate a static universe but a universe with an *accelerated* expansion. In other words, the necessity of a cosmological constant is much more serious because it has not only to counterbalance the gravitational attraction by matter and electromagnetic energy but surpass it and produce an actual universe with negative curvature, i.e., in accelerated expansion. This cosmic repulsion may be understood as a sort of "elasticity" of the spacetime tissue, whose "elastic" energy comes from the zero-point energy of vacuum.

Steven Weinberg was one of the first to express, in a quantitative way, the now famous *cosmological constant problem.* In a paper, published in 1989 [5] and appropriately entitled *"The cosmological constant problem*, he defines the problem and discusses five of its possible theoretical solutions. All of them fails but they play unequivocal roles in the story, that is, they keep burning the flame of hope of a future solution.

Let us state the problem. The cosmological constant was originally inserted in the left-hand side of Einstein's field equation — the metric side —, but it may be in the right-hand side — the energy-momentum side —, and interpreted as a energy component associated to the spacetime substratum, the vacuum. Observational constraints on the accelerated expansion put stringent limits to the vacuum energy density (see [5] for more details). Weinberg calculates the theoretical value of the total energy density of the spacetime substratum by integrating the zero-point energy of all vibrational normal modes associated to the quantum vacuum. He gets an amount that is 10^{118} larger than the "observed" cosmological value. Such a figure follows from a cut-off on the wavenumber of vacuum oscillations, according to the prescriptions of quantum electrodynamics. If one considers only the zero-point energies of quantum chromodynamics the discrepancy is alleviated to a factor of 10^{41}. Which really does not improve much the situation. This is, therefore, the problem. And the second stone.

5 Conclusion

The SMC has an unjustifiable high scientific status. The model has, of course, academic pertinence but it has been *rejected* by the observational and experimental evidences, and hence, it should be considered — on these grounds — as a failed model.

GRT can be evaluated through its two most famous metrics: SM and FRWM. Its overwhelming success is revealed by innumerable and well-succeded applications of SM. Nevertheless it has an equally overwhelming weak point, which is clearly revealed by the FRWM. The SMC, based on FRWM, has demonstrated itself as inconsistent with empirical facts, and became *the uncertain chain that links speculation to speculation in order to prove speculation.* This is too little to proclaim as valid a truly scientific theory. Thus, SMC turned out to be a really heavy stumbling stone to GRT[1].

References

[1] B. Russell, *The ABC of Relativity*, (George Allen & Unwin Ltd., London, 1958).

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[1] In time: 99.999% of the universe is plasma, which turns electromagneto-hydrodynamic interactions into a very important ingredient in its global theoretical description, i.e., in cosmology. This fact is not accounted for in the SMC.

[2] A.K.T. Assis, M.C.D. Neves, D.S.L. Soares, *Hubble's Cosmology: From a Finite Expanding Universe to a Static Endless Universe*, 2nd Crisis in Cosmology Conference, CCC-2. ASP Conference Series, Vol. 413, p.255, Edited by Frank Potter, (Astronomical Society of the Pacific, San Francisco, 2009).

[3] M.S. Turner, *Dark Matter and Dark Energy: The Critical Questions*, `http://arxiv.org/abs/astro-ph/0207297.`

[4] D. Soares, *Einstein's Static Universe*, `http://www.fisica.ufmg.br/~dsoares/ensino/einstt-e.pdf.`

[5] S. Weinberg, *The cosmological constant problem*, 1989, RvMP **61**, pp. 1-23.