

Topics in relativistic cosmology

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Preface

This book is the result of my experience as lecturer of relativistic cosmology at the undergraduate physics course of the Physics Department at the Federal University of Minas Gerais, Belo Horizonte, MG, Brazil, during the period of 2001 to 2010. It is primordially aimed at undergraduate students of physics and astronomy, but readers at the graduate level and researchers in astrophysics and cosmology may find also some benefit reading these topics.

I would like to thank my students of all semesters, during the 10 years I dedicated to the course, for their enthusiastic participation in every activities and for the stimulus they gave me, which led to the compilation of this collection of topics.

Two topics were written in collaboration. The topic about the Friedmann equations was written with my former student Arthur Viglioni and the topic about the correction of the solar motion with my colleague of the Laboratory of Astrophysics Prof. Luiz Paulo Ribeiro Vaz. To both, my sincere thanks. I am particularly grateful also to Prof. Reynier Peletier of the Kapteyn Astronomical Institute, University of Groningen, The Netherlands, for allowing me to use the Kapteyn Institute computer, which was very helpful, especially in the preparation of several figures and diagrams that appear in the topics.

Many of the works undertaken in the courses — including all chapters of this collection — appear on my personal electronic page at the Physics Department (<http://www.fisica.ufmg.br/dsoares/index-e.html>) in the sections “*Topics in relativistic cosmology*” and “*Texts & News*”.

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