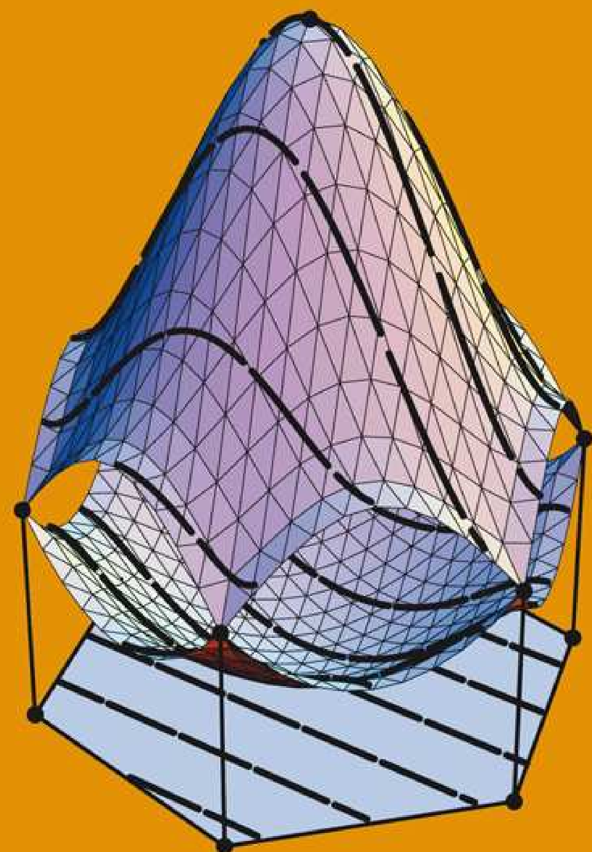

Group Theory

Application to the Physics of Condensed Matter

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Springer

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With 131 Figures and 219 Tables

Springer

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The authors dedicate this book
to John Van Vleck and Charles Kittel

Preface

Symmetry can be seen as the most basic and important concept in physics. Momentum conservation is a consequence of translational symmetry of space. More generally, every process in physics is governed by selection rules that are the consequence of symmetry requirements. On a given physical system, the eigenstate properties and the degeneracy of eigenvalues are governed by symmetry considerations. The beauty and strength of group theory applied to physics resides in the transformation of many complex symmetry operations into a very simple linear algebra. The concept of *representation*, connecting the symmetry aspects to matrices and basis functions, together with a few simple theorems, leads to the determination and understanding of the fundamental properties of the physical system, and any kind of physical property, its transformations due to interactions or phase transitions, are described in terms of the simple concept of symmetry changes.

The reader may feel encouraged when we say group theory is "simple linear algebra." It is true that group theory may look complex when either the mathematical aspects are presented with no clear and direct correlation to applications in physics, or when the applications are made with no clear presentation of the background. The contact with group theory in these terms usually leads to frustration, and although the reader can understand the specific treatment, he (she) is unable to apply the knowledge to other systems of interest. What this book is about is teaching group theory in close connection to applications, so that students can learn, understand, and use it for their own needs.

This book is divided into six main parts. Part I, Chaps. 1–4, introduces the basic mathematical concepts important for working with group theory. Part II, Chaps. 5 and 6, introduces the first application of group theory to quantum systems, considering the effect of a crystalline potential on the electronic states of an impurity atom and general selection rules. Part III, Chaps. 7 and 8, brings the application of group theory to the treatment of electronic states and vibrational modes of molecules. Here one finds the important group theory concepts of *equivalence* and *atomic site* symmetry. Part IV, Chaps. 9 and 10, brings the application of group theory to describe periodic lattices in both real and reciprocal lattices. Translational symmetry gives rise to a linear momentum quantum number and makes the group very large. Here the concepts of *cosets* and *factor groups*, introduced in Chap. 1, are used to factor out the effect of the very large translational group, leading to a finite group to work with each unique type of wave vector – the group of the wave vector. Part V, Chaps. 11–15, discusses phonons and electrons in solid-state physics, considering general positions and specific high symmetry points in the Brillouin zones, and including the addition of spins that have a 4π rotation as the identity transformation. Cubic and hexagonal systems are used as general examples. Finally, Part VI, Chaps. 16–18, discusses other important symmetries, such as time reversal symmetry, important for magnetic systems, permutation groups, important for many-body systems, and symmetry of tensors, important for other physical properties, such as conductivity, elasticity, etc.

This book on the application of Group Theory to Solid-State Physics grew out of a course taught to Electrical Engineering and Physics graduate students by the authors and developed over the years to address their professional needs. The material for this book originated from group theory courses taught by Charles Kittel at U.C. Berkeley and by J.H. Van Vleck at Harvard in the early 1950s and taken by G. Dresselhaus and M.S. Dresselhaus, respectively. The material in the book was also stimulated by the classic paper of Bouckaert, Smoluchowski, and Wigner [1], which first demonstrated the power of group theory in condensed matter physics. The diversity of applications of group theory to solid state physics was stimulated by the research interests of the authors and the many students who studied this subject matter with the authors of this volume. Although many excellent books have been published on this subject over the years, our students found the specific subject matter, the pedagogic approach, and the problem sets given in the course user friendly and urged the authors to make the course content more broadly available.

The presentation and development of material in the book has been tailored pedagogically to the students taking this course for over three decades at MIT in Cambridge, MA, USA, and for three years at the University Federal of Minas Gerais (UFMG) in Belo Horizonte, Brazil. Feedback came from students in the classroom, teaching assistants, and students using the class notes in their doctoral research work or professionally.

We are indebted to the inputs and encouragement of former and present students and collaborators including, Peter Asbeck, Mike Kim, Roosevelt Peoples, Peter Eklund, Riichiro Saito, Georgii Samsonidze, Jose Francisco de Sampaio, Luiz Gustavo Cançado, and Eduardo Barros among others. The preparation of the material for this book was aided by Sharon Cooper on the figures, Mario Hofmann on the indexing and by Adelheid Duhm of Springer on editing the text. The MIT authors of this book would like to acknowledge the continued long term support of the Division of Materials Research section of the US National Science Foundation most recently under NSF Grant DMR-04-05538.

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August 2007 *Ado Jorio*

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