

B

Two-Dimensional Space Groups

We include in this appendix a summary of the crystallographic symmetries for all 17 of the 2D space groups, taken from the “International Tables for X-ray Crystallography” [58].

Table B.1. The two-dimensional oblique space group $p1$ or #1 ($p1$)

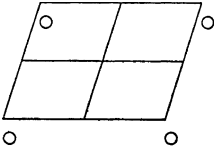
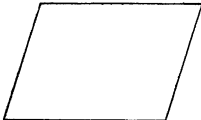
$p1$	No. 1	$p1$	1 Oblique
			
	Origin on 1		
Number of positions	Co-ordinates of	Conditions limiting	
Wyckoff notation,	equivalent positions	possible reflections	
and point symmetry		General: No conditions	
1 a 1 x, y			

Table B.2. The two-dimensional oblique space group $p2$ or #2 ($p211$)

$p2$	No. 2	$p211$	2 Oblique	
Origin at 2				
2	e	1	$x, y; \bar{x}, \bar{y}$	General: No conditions
1	d	2	$\frac{1}{2}, \frac{1}{2}$	Special: No conditions
1	c	2	$\frac{1}{2}, 0$	
1	b	2	$0, \frac{1}{2}$	
1	a	2	$0, 0$	

Table B.3. The two-dimensional rectangular space group pm or #3 ($p1m1$)

pm	No. 3	$p1m1$	m Rectangular	
Origin on m				
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections		
2	c	1	$x, y; \bar{x}, y$	General: No conditions
1	b	m	$\frac{1}{2}, y$	Special:
1	a	m	$0, y$	No conditions

Table B.4. The two-dimensional space group pg or #4 ($p1g1$)

pg	No. 4	$p1g1$	m Rectangular
	Origin on g		
2	a	1	$x, y; \bar{x}, \frac{1}{2} + y$
			General: hk : No conditions $0k$: $k = 2n$

Table B.5. The two-dimensional rectangular space group cm or #5 ($c1m1$)

cm	No. 5	$c1m1$	m Rectangular
	Origin on m		
Number of positions	Co-ordinates of	Conditions limiting	
Wyckoff notation,	equivalent positions	possible reflections	
and point symmetry	$(0, 0; \frac{1}{2}, \frac{1}{2}) +$		
4	b	1	$x, y; \bar{x}, y$
			General: hk : $h + k = 2n$
2	a	m	Special: as above only

Table B.6. The two-dimensional rectangular space group pmm or #6 ($p2mm$)

pmm	No. 6	$p2mm$	mm Rectangular
Origin at $2mm$			
Number of positions	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
Wyckoff notation, and point symmetry			
4 i 1 $x, y; \bar{x}, y; \bar{x}, \bar{y}; x, \bar{y}$		General: no conditions	
2 h m $\frac{1}{2}, y; \frac{1}{2}, \bar{y}$		Special: No condition	
2 g m $0, y; 0, \bar{y}$			
2 f m $x, \frac{1}{2}; \bar{x}, \frac{1}{2}$			
2 e m $x, 0; \bar{x}, 0$			
1 d mm $\frac{1}{2}, \frac{1}{2}$			
1 c mm $\frac{1}{2}, 0$			
1 b mm $0, \frac{1}{2}$			
1 a mm $0, 0$			

Table B.7. The two-dimensional rectangular space group pmg or #7 ($p2mg$)

pmg	No. 7	$p2mg$	mm Rectangular
Origin at 2			
Number of positions	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
Wyckoff notation, and point symmetry			
4 d 1 $x, y; \bar{x}, \bar{y}; \frac{1}{2} + x, \bar{y}; \frac{1}{2} - x, y$		General: hk : No conditions $h0$: $h = 2n$	
2 c m $\frac{1}{4}, y; \frac{3}{4}, \bar{y}$		Special: as above, plus no extra conditions	
2 b 2 $0, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}$		} hk : $h = 2n$	
2 a 2 $0, 0; \frac{1}{2}, 0$			

Table B.8. The two-dimensional rectangular space group pgg or #8 ($p2gg$)

pgg	No. 8	$p2gg$	mm Rectangular
Origin at 2			
Number of positions	Co-ordinates of	Conditions limiting	
Wyckoff notation,	equivalent positions	possible reflections	
and point symmetry		General:	
4 c 1 $x, y; \bar{x}, \bar{y}; \frac{1}{2} + x, \frac{1}{2} - y; \frac{1}{2} - x, \frac{1}{2} + y$		hk : no conditions	
		$h0$: $h = 2n$	
		$0l$: $k = 2n$	
		Special: as above, plus	
2 b 2 $\frac{1}{2}, 0; 0, \frac{1}{2}$		} hk : $h + k = 2n$	
2 a 2 $0, 0; \frac{1}{2}, \frac{1}{2}$			

Table B.9. The two-dimensional rectangular space group cmm or #9 ($c2mm$)

cmm	No. 9	$c2mg$	mm Rectangular
Origin at $2mm$			
Number of positions	Co-ordinates of	Conditions limiting	
Wyckoff notation,	equivalent positions	possible reflections	
and point symmetry	$(0, 0; \frac{1}{2}, \frac{1}{2}) +$	General:	
8 f 1 $x, y; \bar{x}, y; \bar{x}, \bar{y}; x, \bar{y}$		hk : $h + k = 2n$	
		Special: as above, plus	
4 e m $0, y; 0, \bar{y}$		} no extra conditions	
4 d m $x, 0; \bar{x}, 0$			
4 c 2 $\frac{1}{4}, \frac{1}{4}; \frac{1}{4}, \frac{3}{4}$		hk : $h = 2n; (k = 2n)$	
2 b mm $0, \frac{1}{2}$		} no extra conditions	
2 a mm $0, 0$			

Table B.10. The two-dimensional square space group $p4$ or #10 ($p4$)

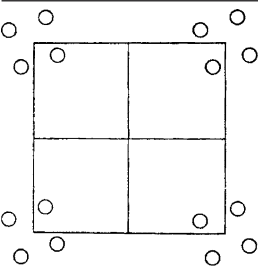
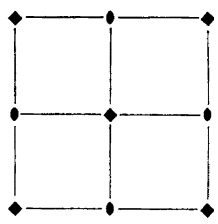
$p4$	No. 10	$p4$	4 Square
			
	Origin at 4		
Number of positions	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
Wyckoff notation, and point symmetry			
4 d 1 $x, y; \bar{x}, \bar{y}; y, \bar{x}; \bar{y}, x$		General: No conditions	
2 c 2 $\frac{1}{2}, 0; 0, \frac{1}{2}$		Special: $hk: h + k = 2n$	
1 b 4 $\frac{1}{2}, \frac{1}{2}$		} No conditions	
1 a 4 $0, 0$			

Table B.11. The two-dimensional square space group $p4m$ or #11 ($p4mm$)

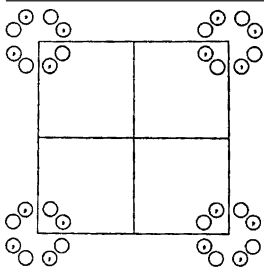
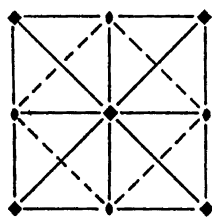
$p4m$	No. 11	$p4mm$	$4mm$ Square
			
Origin at $4mm$			
Number of positions	Co-ordinates of	Conditions limiting	
Wyckoff notation,	equivalent positions	possible reflections	
and point symmetry			
8 g 1	$x, y; \bar{x}, \bar{y}; y, \bar{x}; \bar{y}, x; \bar{x}, y; x, \bar{y}; \bar{y}, \bar{x}; y, x$	General:	
		No conditions	
		Special:	
4 f m	$x, x; \bar{x}, \bar{x}; \bar{x}, x; x, \bar{x}$	} no conditions	
4 e m	$x, \frac{1}{2}; \bar{x}, \frac{1}{2}; \frac{1}{2}, x; \frac{1}{2}, \bar{x}$		
4 d m	$x, 0; \bar{x}, 0; 0, x; 0, \bar{x}$	} $hk: h + k = 2n$	
2 c mm	$\frac{1}{2}, 0; 0, \frac{1}{2}$		
1 b $4mm$	$\frac{1}{2}, \frac{1}{2}$	} no conditions	
1 a $4mm$	$0, 0$		

Table B.12. The two-dimensional square space group $p4g$ or #12 ($p4gm$)

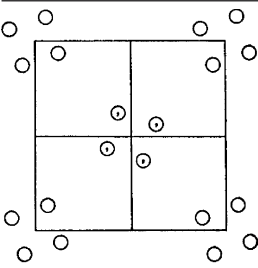
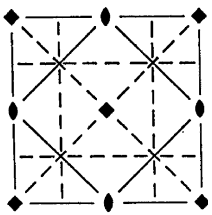
$p4g$	No. 12	$p4gm$	$4mm$ Square
			
Origin at 4			
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
General:			
8 d 1	$x, y; y, \bar{x}; \frac{1}{2} - x, \frac{1}{2} + y; \frac{1}{2} - y, \frac{1}{2} - x$ $\bar{x}, \bar{y}; \bar{y}, x; \frac{1}{2} + x, \frac{1}{2} - y; \frac{1}{2} + y, \frac{1}{2} + x$	hk : No conditions $h0$: $h = 2n$ ($0k$: $k = 2n$) hh : No conditions Special: as above, plus no extra conditions	
4 c m	$x, \frac{1}{2} + x; \bar{x}, \frac{1}{2} - x; \frac{1}{2} + x, \bar{x}; \frac{1}{2} - x, x$	} hk : $h + k = 2n$	
2 b $4mm$	$\frac{1}{2}, 0; 0, \frac{1}{2}$		
2 a 4	$0, 0; \frac{1}{2}, \frac{1}{2}$		

Table B.13. The two-dimensional hexagonal space group $p3$ or #13 ($p3$)

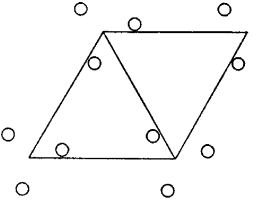
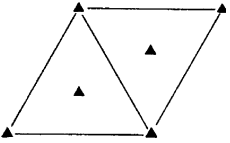
$p3$	No. 13	$p3$	3 Hexagonal
			
Origin at 3			
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
3 d 1	$\bar{y}, x - y; y - x, \bar{x}$	General: No conditions Special: no conditions	
1 c 3	$\frac{1}{3}, \frac{1}{3}$		
1 b 3	$\frac{1}{3}, \frac{1}{3}$		
1 a 3	$0, 0$		

Table B.14. The two-dimensional hexagonal space group $p3m1$ or #14 ($p3m1$)

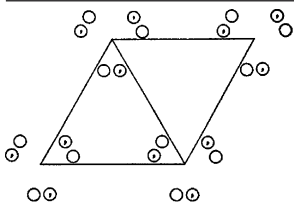
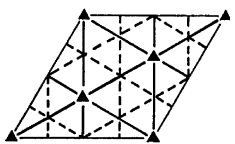
$p3m1$	No. 14	$p3m1$	$3m$ Hexagonal
			
Origin at $3m1$			
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
6 e m $x, y; \bar{y}, x - y; y - x, \bar{x}$ $\bar{y}, \bar{x}; x, x - y; y - x, y$		General: No conditions	
3 d m $x, \bar{x}; x, 2x; 2\bar{x}, x$		Special: No conditions	
1 c $3m$ $\frac{2}{3}, \frac{1}{3}$			
1 b $3m$ $\frac{1}{3}, \frac{2}{3}$			
1 a $3m$ $0, 0$			

Table B.15. The two-dimensional hexagonal space group $p31m$ or #15 ($p31m$)

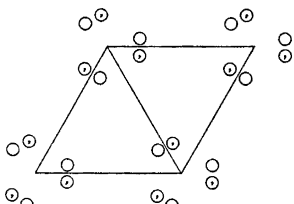
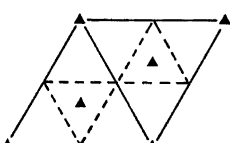
$p31m$	No. 15	$p31m$	$3m$ Hexagonal
			
Origin at $31m$			
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
6 d 1 $x, y; \bar{y}, x - y; y - x, \bar{x}$ $y, x; \bar{x}, y - x; x - y, \bar{y}$		General: No conditions	
3 c m $x, 0; 0, x; \bar{x}, \bar{x}$		Special: no conditions	
2 b 3 $\frac{1}{3}, \frac{2}{3}; \frac{2}{3}, \frac{1}{3}$			
1 a $3m$ $0, 0$			

Table B.16. The two-dimensional hexagonal space group $p6$ or #16 ($p6$)

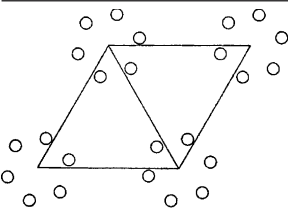
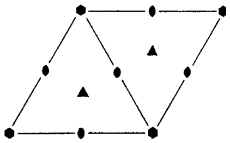
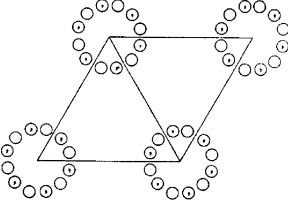
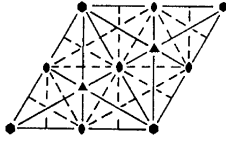
$p6$	No. 16	$p6$	Hexagonal 6
			
Origin at 6			
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
6 d 1	$x, y; \bar{y}, x - y; y - x, \bar{x}$ $\bar{x}, \bar{y}; y, y - x; x - y, x$	General: No conditions	
3 c 2	$\frac{1}{2}, 0; 0, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}$	Special: No conditions	
2 b 3	$\frac{1}{3}, \frac{2}{3}; \frac{2}{3}, \frac{1}{3}$		
1 a 6	0, 0		

Table B.17. The two-dimensional hexagonal space group $p6m$ or #17 ($p6mm$)

$p6m$	No. 17	$p6m$	$6mm$ Hexagonal
			
Origin at $6mm$			
Number of positions Wyckoff notation, and point symmetry	Co-ordinates of equivalent positions	Conditions limiting possible reflections	
12 f 1	$x, y; \bar{y}, x - y; y - x, \bar{x}; y, x; \bar{x}, y - x; x - y, \bar{y}$ $\bar{x}, \bar{y}; y, y - x; x - y, x; \bar{y}, \bar{x}; x, x - y; y - x, y$	General: No conditions Special: No conditions	
6 e m	$x, \bar{x}; x, 2x; 2\bar{x}, \bar{x}; \bar{x}, x; \bar{x}, 2\bar{x}; 2x, x$		
6 d m	$x, 0; 0, x; \bar{x}, \bar{x}; \bar{x}, 0; 0, \bar{x}; x, x$		
3 c mm	$\frac{1}{2}, 0; 0, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}$		
2 b $3m$	$\frac{1}{3}, \frac{2}{3}; \frac{2}{3}, \frac{1}{3}$		
1 a $6mm$	0, 0		