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Limb-darkening coefficients of illuminated atmospheres

I. Results for illuminated line-blanketed models with $3\,700\,\mathrm{K} < T_{\mathrm{eff}} < 7\,000\,\mathrm{K}^{\star}$

S.H.P. Alencar[1] **and L.P.R. Vaz**[1]

Departamento de Física, ICEX–UFMG, C.P. 702, 30.123-970 Belo Horizonte, MG, Brazil

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Abstract. The effect of mutual illumination in a close binary on the limb-darkening coefficients is studied using the UMA (Uppsala Model Atmosphere) code in convective line-blanketed atmospheres illuminated by line-blanketed fluxes, for $3\,700\,\mathrm{K} < T_{\mathrm{eff}} < 7\,000\,\mathrm{K}$ and $7\,000\,\mathrm{K} < T_{\mathrm{h}} < 12\,000\,\mathrm{K}$. The results show that the limb-darkening coefficients of illuminated atmospheres are significantly different from the non-illuminated ones and mostly depend on four parameters: the amount of incident energy, the illumination direction, the effective temperature of the illuminated star and the temperature of the illuminating one. We present bolometric, monochromatic and passband specific coefficients and we give, for the latter, analytical expressions in order to easily account for the effect in light curve synthesis programs.

Key words: stars: fundamental parameters — stars: atmospheres — binaries: close; eclipsing

1. Introduction

Limb-darkening coefficients can only be directly determined for very few stars apart from the Sun. Due to that, when computing light curves (LC) synthesis programs, those coefficients are usually interpolated from tables of theoretical values calculated from atmosphere models.

Over the past decades many analytical approximations have been proposed to describe the variation of the intensity over a stellar surface. Initially, the most adopted was the linear limb-darkening law (Milne 1921):

$$R_\lambda(\mu) = \frac{I_\lambda(\mu)}{I_\lambda(1)} = 1 - x_\lambda(1-\mu) \tag{1}$$

where I_λ is the beam intensity at the wavelength λ, μ is the co-sinus of γ, the angle between the atmosphere normal and the beam direction (the line of sight angle, see Fig. 1) and x_λ is the so-called limb-darkening coefficient. This law gives a good description of the limb-darkening in the solar atmosphere that, due to its temperature and physical conditions, is reasonably well represented by a grey atmosphere (for which the limb-darkening is well adjusted by the linear law). Lately, mainly due to theoretical studies on stellar atmospheres, other non-linear laws were proposed, as the polynomial approximations, that better described the effect away from the solar temperature range:

$$R_\lambda(\mu) = \frac{I_\lambda(\mu)}{I_\lambda(1)} = 1 - x_\lambda(1-\mu) - y_\lambda(1-\mu)^n \tag{2}$$

where y_λ are the non-linear limb-darkening coefficients, $n = 2$ (Wade & Ruciński 1985, using the version 1979 of ATLAS by Kurucz 1979; Manduca et al. 1977 and Claret & Giménez 1990, using versions of UMA, see Gustafsson et al. 1975; Bell et al. 1976) for the quadratic case and $n = 3$ (Van't Veer 1960, quoted by Díaz-Cordovés & Giménez 1992) for the cubic law.

The logarithmic approximation, proposed by Klingesmith & Sobieski (1970) gave very good results in representing their theoretical models, valid for the interval $10\,000\,\mathrm{K} < T_{\mathrm{eff}} < 40\,000\,\mathrm{K}$:

$$R_\lambda(\mu) = \frac{I_\lambda(\mu)}{I_\lambda(1)} = 1 - x_\lambda(1-\mu) - y_\lambda \mu \ln \mu, \tag{3}$$

confirmed by Van Hamme (VH, 1993), with ATLAS (version 1991). Díaz-Cordovés & Giménez (1992, using ATLAS version 1979) proposed a new non-linear approximation, a square root limb-darkening law:

$$R_\lambda(\mu) = \frac{I_\lambda(\mu)}{I_\lambda(1)} = 1 - x_\lambda(1-\mu) - y_\lambda(1-\sqrt{\mu}). \tag{4}$$

Send offprint requests to: L.P.R. Vaz
e-mails: silvia@fisica.ufmg.br, lpv@fisica.ufmg.br

* Tables 1, 2 and 3 will be also accessible in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/Abstract.html

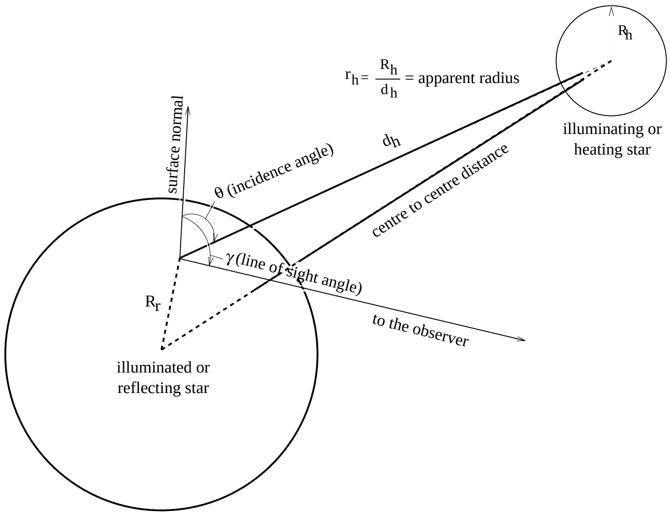

Fig. 1. Geometry of illumination in a double star system

In subsequent papers (Claret et al. 1995 and Díaz-Cordovés et al. 1995, both using ATLAS version 1991), studies are made only with the square root law and the quadratic and linear ones. In that case, the square root approximation seems almost always to be the best one.

According to VH, the logarithmic law gives the best approximation in the UV, while the square root law is the best in the IR and longer wavelengths. In the optical region, cooler stars are better represented by the logarithmic law and high temperature stars by a square root law.

In close binary systems, the mutual irradiation affects the light curves and the spectra of both stars. The reflection effect present in close binary systems has a strong influence on limb-darkening coefficients, as already noticed by Vaz & Nordlund (1985), Nordlund & Vaz (1990) and Claret & Giménez (CG, 1990). One effect concerns the values of the limb-darkening coefficients, which are changed by the infalling flux. In fact, even the limb-darkening law which best represents the variation of the flux with the line of sight angle may change due to the external illumination. Another important difference, as compared to the normal stellar atmospheres (for which the limb-darkening laws and coefficients are valid over the whole stellar surface), is that the illuminated atmospheres show different limb-darkening coefficients (and laws) for different points on the stellar surface, due to the dependence of these on the incidence angle of the infalling flux. Therefore, the best representation for the center-to-limb variation of the surface brightness of eclipsing binary components in the synthetic LC generation studies is to use the coefficients calculated for the local configuration (i.e. considering the apparent radius, the direction of illumination and the temperature of the companion, see below).

Our goal is to understand how the external illumination affects the limb-darkening laws and coefficients and to present the results in a way to easily account for the effect in LC synthesis programs. We made calculations of bolometric, passband-specific and monochromatic limb-darkening coefficients for all the laws presented above.

In real systems both stars are affected by the mutual illumination. However, in the present study we do not consider the effect of the irradiation from the illuminated (or reflecting) star on the illuminating one (also referred to as heating or source star), i.e. the small changing in the effective temperature of the illuminating star due to the "reflection effect" from its companion. As we are considering mostly systems for which the illuminated star is cooler than the illuminating one, this is probably a minor contribution. Consequently, we did not take second order effects into account (back reflection), either.

In Sect. 2 we describe the method and study the effect of illumination on the limb-darkening laws and coefficients (bolometric, monochromatic and for the passband specific filters of the Strömgren and of the Johnson-Morgan photometric systems), presenting the results as polynomial expressions and tables. We discuss the results in Sect. 3, and present our conclusions in Sect. 4.

2. The atmosphere model

We use the Uppsala Model Atmosphere (UMA, Gustafsson et al. 1975; Bell et al. 1976) code, in a version by Vaz & Nordlund (1985), as described in Alencar & Vaz (1997). The code is intended for cool ($3\,500\,\mathrm{K} < T_{\rm eff} < 8\,000\,\mathrm{K}$) atmospheres, and assumed hydrostatic equilibrium, a plane–parallel structure, and local thermodynamic equilibrium. Convection is modelled with the mixing length recipe.

An illuminated model is defined by its effective temperature $T_{\rm eff}$, its surface acceleration of gravity g, mixing length parameter $\alpha = l/H_P$, and chemical composition (fixed at solar abundance in this work). The external illumination is parametrized by the direction of illumination ν (the cosine of θ, the incidence angle with respect to the surface normal), the effective temperature of the heating star $T_{\rm h}$, and its apparent radius $r_{\rm h}$ (the ratio between the radius of the heating star and the distance from its centre to the point on the surface of the reflecting star). The geometry of illumination in a double star system is shown in Fig. 1. We kept $\alpha = l/H_P$ fixed at 1.5 for the reflecting star atmosphere.

We noticed that the effect caused by illumination on the limb-darkening was rather weakly dependent on the log g of the model. This can be seen in Fig. 2, where we show that the effect of changing the $\log g$ from 3.5 to 5.0 corresponds to only $\pm 7\%$ and $\pm 9\%$ of the overall effect of illuminating an atmosphere of $T_{\rm eff} = 3\,697\,\mathrm{K}$ by $T_{\rm h} = 3\,700\,\mathrm{K}$ and $10\,000\,\mathrm{K}$, respectively. Therefore, we did calculations only for log $g = 4.5$, as for this value we achieve a representative mean effect of illumination for the interval $3.5 \leq \log\ g \leq 5.0$. We study illuminated atmospheres with line absorption (line-blanketed or ODF)

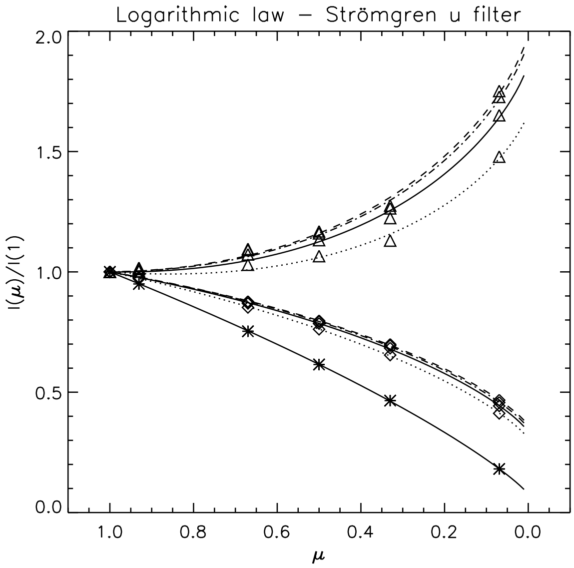

Fig. 2. Comparison between the effect of the surface gravity and of external illumination on the limb-darkening, here adjusted by the logarithmic law, see Eq. (3). The * symbols correspond to the non–illuminated model with $T_{\rm eff} = 3\,697$ K. The $\diamond$ and $\triangle$ symbols correspond to this model illuminated by $T_{\rm h} = 3\,700$ K and $10\,000$ K, respectively. The dashed lines correspond to models with $\log g = 3.5$, the dash-dotted ones to $\log g = 4.0$, the solid lines to $\log g = 4.5$ and the dotted lines to $\log g = 5.0$

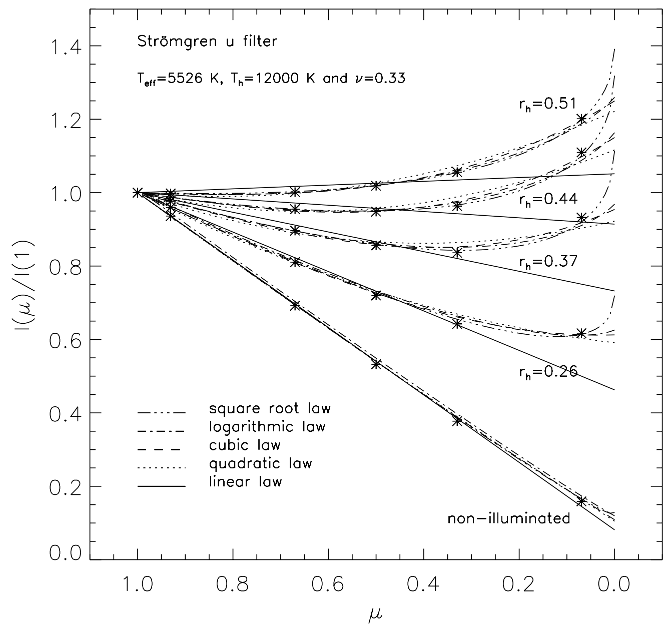

Fig. 3. Adjusted limb-darkening laws for a non-illuminated model and 4 illuminated models (varying $r_{\rm h}$). The asterisks represent the model results

in convective equilibrium. The ODF tables limit the temperature range of this study to those corresponding to (ZAMS) stars with masses ranging from $0.6\,M_\odot$ to $1.5\,M_\odot$.

This same model has been used to study, in another paper and for the first time in the literature, the effect of illumination on the gravity brightening exponent β, another important parameter for LC synthesis. That work (Alencar et al. 1998) is a continuation of the study of the β exponent using stellar atmospheres (Alencar & Vaz 1997).

2.1. The method

The atmosphere model generates intensities in six different angular directions ($\mu = 0.06943, 0.33001, 0.5, 0.66999, 0.93057$ and 1.0) and 368 wavelengths ranging from 1500 Å to 124000 Å. Starting from these data, monochromatic, passband-specific and bolometric limb-darkening coefficients can be calculated. We will present here the results obtained using two different calculation methods. The first one, described by CG, determines the coefficients by least square-fitting of the integrated and normalized model intensities. The second method follows the procedure outlined by VH where a number of physical constraints, equal to the number of coefficients to be determined, are assumed. The limb-darkening coefficients are obtained by solving the constraint equations. Using a one parameter law, the constraint is the conservation of the total flux and for a two parameter law, the additional constraint that the mean intensity of the approximation and the atmosphere model must be equal is applied. When calculating the passband coefficients we made a convolution of the intensities with the response functions corresponding to the Strömgren filters *uvby* (Crawford & Barnes 1970) and the *UBVRI* passbands (Bessel 1983). We used in the convolution the atmospheric mean transmission by Allen (1976), the reflection curve of two aluminum coated mirrors (Allen 1976), the sensitivity of a 1P21 photo-multiplier from Kurucz (1979) and for the Strömgren filters, the sensitivity function of the SAT photometer (Florentin-Nielsen 1983, personal communication).

We calculated models with relative fluxes ($F_{{\rm rel},\nu} = [T_{\rm h}/T_{\rm eff}]^4 {r_{\rm h}}^2\nu$) ranging from 0 to 2, whenever possible ($r_{\rm h} < 1$). Those values are easily found in the literature amongst many types of binary systems. Using $\nu = 1$ we find $F_{\rm rel} = 0.446$ for V Pup (detached, Andersen et al. 1983), 0.338 for LZ Cen (detached, Vaz et al. 1995), 0.386 for RY Aqr (semi–detached, Helt 1987), 0.660 for DH Cep (detached, Hilditch et al. 1996), 1.35 for AT Peg (algol, Maxted et al. 1994), 2.39 for HU Tau (algol, Maxted et al. 1995) and 2.72 for TV Cas (algol, Khalesseh & Hill 1992).

Many of the binary systems above also have temperatures that fall in our selected ranges: RY Aqr ($T_{\rm eff} = 4\,500$ K and $T_{\rm h} = 7\,600$ K), AT Peg (4 898 K and 8 395 K), HU Tau (5 495 K and 12 022 K) and TV Cas (5 248 K and 10 471 K).

As with $T_{\rm h}$ close to $T_{\rm eff}$ we could not get large values of $F_{\rm rel}$ with reasonable values of $r_{\rm h}$, we studied models mostly ranging in the interval $7\,000\,{\rm K} < T_{\rm h} < 12\,000\,{\rm K}$.

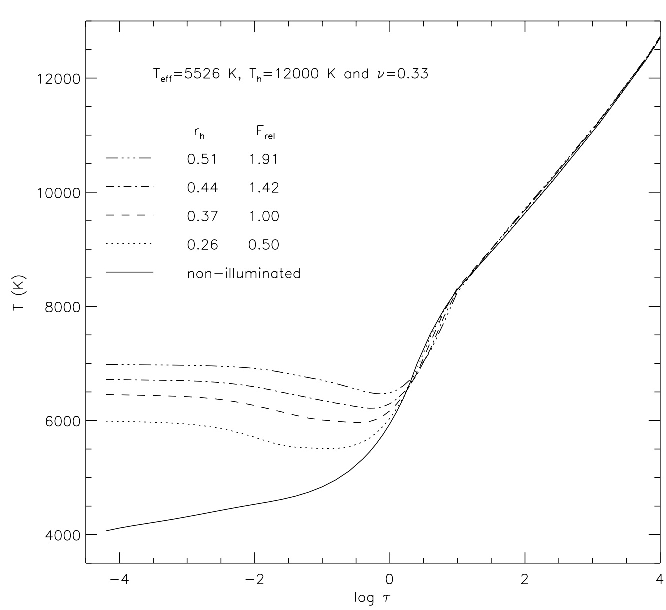

Fig. 4. The run of the temperature with the optical depth (Rosseland mean) for the models shown in Fig. 3

The illuminating non–grey fluxes with $T_{\mathrm{h}} \leq 7\,000\,\mathrm{K}$ were generated with the UMA code, while for higher T_{h} we took the fluxes from Kurucz (1979). We did calculate some models with T_{h} close to T_{eff} ($T_{\mathrm{h}} = 3700\,\mathrm{K}$, $4600\,\mathrm{K}$, $5500\,\mathrm{K}$, $6700\,\mathrm{K}$) and the results are in agreement with our studies with higher T_{h} so that our results do apply for lower heating temperatures, also. However, we advise caution in using those approximations extended to outside the limits proposed in this work.

In Sect. 2.2 we give the results obtained with line-blanketed atmospheres illuminated by line-blanketed fluxes, showing differences in the calculations with the two distinct methods.

2.2. Results

Figure 3 shows that the external illumination strongly affects the limb-darkening laws and coefficients of illuminated atmospheres as compared with the non-illuminated ones. These models are approximately equivalent to those calculated by CG and ν has a mean value between vertical and grazing incidence. In order to be consistent we will show the results for the Strömgren u filter, but the effects are similar for the other calculations done, bolometric, monochromatic and for all the Strömgren filters and the $UBVRI$ filters of the Johnson-Morgan system, as well. As we increase the amount of incident energy, for example by increasing the apparent radius while keeping the other parameters fixed, the law for non-illuminated models no longer represents the calculated intensities, showing that non-illuminated coefficients yield wrong limb-darkening laws when used with illuminated stars.

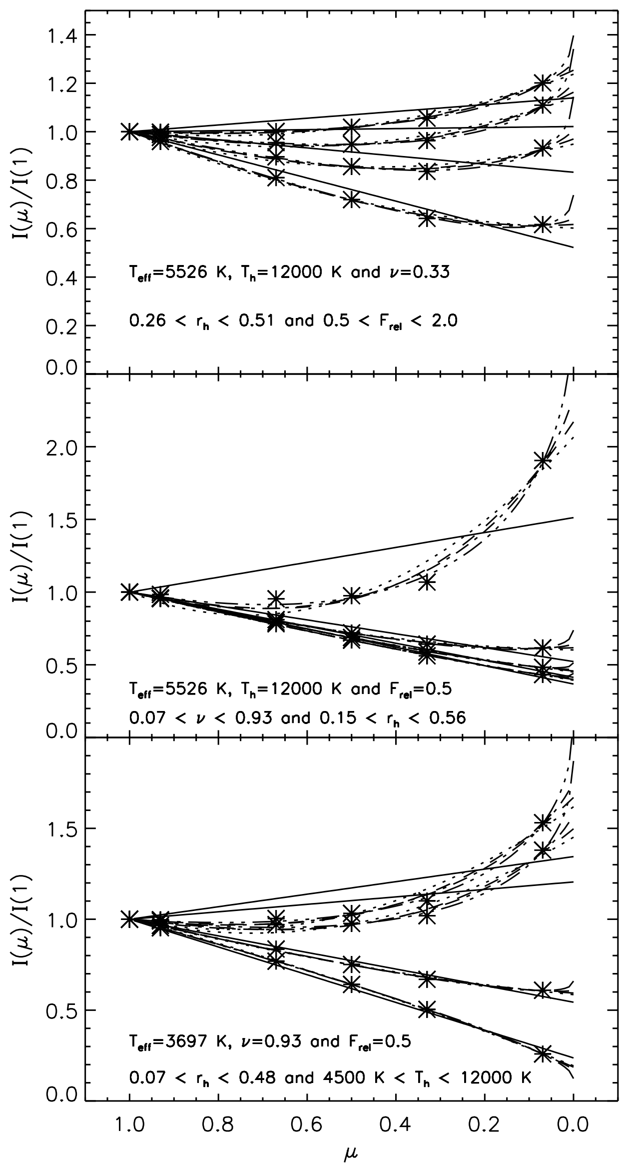

Fig. 5. $I(\mu)/I(1)$ vs. μ curves for the Strömgren u filter and various models, see the text for explanation. The meaning of the different linestyles and symbols is the same as in Fig. 3

A first interesting result from Fig. 3 is that, depending on the set of parameters chosen, we observe an effect of limb brightening instead of darkening. This can be understood as follows: when looking at a stellar disk we normally see the limb darker than the center because, despite of seeing the same optical depth in both, it corresponds to deeper, and consequently hotter, layers in the center than in the limb. In a illuminated star, the temperature distribution in the atmosphere becomes more and more homogeneous as we increase the illumination flux, until it reaches a state where there is no center-to-limb darkening and the star's brightness is the same throughout the stellar disk. This can be seen in Fig. 4, where we show the

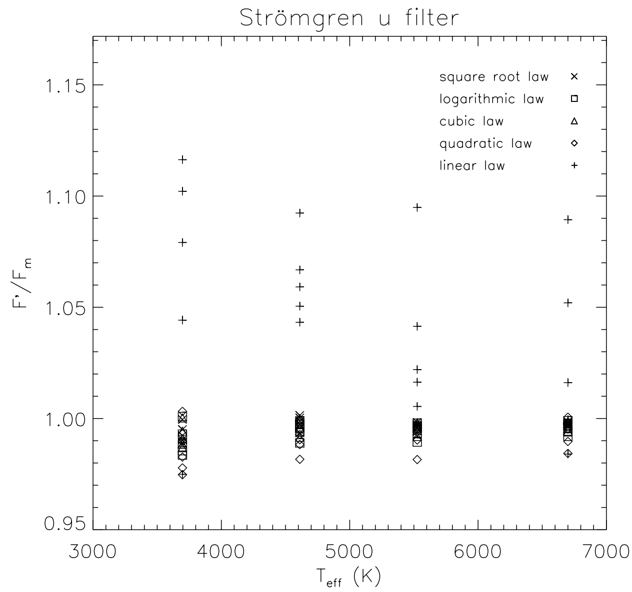

Fig. 6. $F'/F_{\rm m}$ calculated with the coefficients determined by the CG method. Models used: $\nu = 0.07$, $T_{\rm h} = 7000$ K

temperature structure for the models shown in Fig. 3. The larger the amount of infalling flux the higher the temperature of the external layers and the larger the region where it stays approximately constant.

If we keep increasing $F_{\rm rel}$, by changing either $r_{\rm h}$, $T_{\rm h}$ or ν, we reach a situation where the most external layers are hotter than the internal ones. As we see more external layers looking at the limb than looking at the center, we have a limb brightening effect. In Fig. 5 we illustrate that effect in three panels, each with four $I(\mu)/I(1)$ vs. μ curves, varying the many parameters. The effect is, though, dependent not only on the amount of the infalling flux. The first panel of Fig. 5 shows models that reach limb brightening with constant $T_{\rm h}$ and incident direction, but changing $F_{\rm rel}$ through changes in $r_{\rm h}$. Panels 2 and 3 show that limb brightening can also be achieved with constant $F_{\rm rel}$ $(= 0.5)$, either by keeping $T_{\rm h}$ constant and using different pairs of $r_{\rm h}$ and ν, or by fixing ν and changing $r_{\rm h}$ and $T_{\rm h}$. We can also notice in Figs. 3 and 5 that the linear law (straight lines) is a poor approximation for the limb-darkening of illuminated atmospheres, while for $T_{\rm eff} \approx 5\,500$ K it fits well the non-illuminated case (Fig. 3). One should mention here that, although the models of Fig. 3 are similar to those calculated by CG, they do not report any evidence for the limb brightening effect in their work.

The methods by CG and VH, used here to calculate the coefficients, differ from each other in some important aspects. While CG emphasize the point of using coefficients that best match the relation $I(\mu)/I(1)$ vs. μ, even if the total flux is not perfectly conserved, VH says that the physical constraints of his method are introduced in order to avoid non-physical coefficients. In non-illuminated atmospheres CG find that the total emergent flux obtained by integrating the intensities calculated from the adjusted coefficients (F') does not differ more than 2% from the total flux obtained from the model intensities $(F_{\rm m})$, and that when choosing a non-linear law, the difference vanishes. We calculated the relation $(F'/F_{\rm m})$ for all our illuminated models, and the result for a set of models is shown in Fig. 6. As noticed, the linear law is not a good approximation when dealing with illuminated atmospheres. We find that the mean difference between the fluxes is around 4% (with a maximum of 26% in some models) in the case of a linear law and less than 1% (however with a maximum of 6%) for the non-linear ones. These results show that for an illuminated atmosphere the CG method should be used with care, as, for some models, even choosing a non-linear law, the flux is not conserved. Due to that we decided to adopt the VH method in our calculations of all the results presented in Tables 1 and 2.

Tables 1 and 2 are a sample of the tables with the monochromatic, bolometric and passband-specific limb-darkening coefficients than can be accessed electronically. We calculated coefficients for the linear, quadratic, cubic, logarithmic and square root laws, for stars with $3700\,{\rm K} \le T_{\rm eff} \le 6700\,{\rm K}$ heated by a companion with $7000\,{\rm K} \le T_{\rm h} \le 12000\,{\rm K}$, with relative fluxes ranging from 0.0 to 2.0 and the illumination direction, ν, varying from 0.07 to 0.97. The parameter Q in Table 1 has the same meaning as in VH. It is a quality factor that shows how good is the fitting to the model intensities, the smaller the Q the better the fitting:

$$Q_\lambda = \sqrt{\frac{\sum_{i=1}^{6}[R_\lambda(\mu_i) - \widehat{R_\lambda}(\mu_i)]^2}{6-m}} \tag{5}$$

where $m = 1$ for a one-parameter law (linear law) and $m = 2$ for a two parameter law (non-linear laws). $R_\lambda(\mu)$ stands for the ratio $I_\lambda(\mu)/I_\lambda(1)$ determined with the atmosphere model and $\widehat{R_\lambda}(\mu)$ for the same ratio determined with the limb-darkening approximation. When determining the bolometric coefficient, Q was calculated as:

$$Q = \frac{\int_{\lambda_i}^{\lambda_f} F(\lambda) Q(\lambda)}{\int_{\lambda_i}^{\lambda_f} F(\lambda)} \tag{6}$$

where $F(\lambda)$ is the monochromatic flux, λ_i is the shortest and λ_f the longest wavelength used in the UMA code.

When calculating with the CG method we do not determine the monochromatic limb-darkening coefficients, so Q cannot be obtained by integrating them. In order to be able to compare the results from the CG and VH methods, we used in Table 2, the following similar definition:

$$Q_{\rm filter} = \sqrt{\frac{\sum_{i=1}^{6}[R_{\rm filter}(\mu_i) - \widehat{R_{\rm filter}}(\mu_i)]^2}{6-m}}. \tag{7}$$

Although only the VH results are presented in Table 2, we performed calculations with both methods. In general, the CG method gives a smaller Q than VH, representing

Table 1. Sample of the table with the bolometric and monochromatic limb-darkening coefficients

Model 5, $T_{\rm eff}$ = 3697 K, $F_{\rm rel}$ = 0.150, ν = 0.06943, $T_{\rm h}$ = 7000 K

	Linear Law		Quadratic Law			Cubic Law			Logarithmic Law			Square Root Law			$I(\lambda,\mu=1)$
λ (nm)	x	Q	x	y	Q	x	y	Q	x	y	Q	x	y	Q	$\left(\frac{\rm erg}{\rm s\,cm^2\,ster\,nm}\right)$
bolometric	0.240	(0.0300)	0.221	0.040	(0.0115)	0.230	0.033	(0.0088)	0.267	0.040	(0.0088)	0.181	0.099	(0.0080)	
153.20	−0.099	(0.0128)	−0.130	0.062	(0.0023)	−0.114	0.051	(0.0007)	−0.058	0.062	(0.0008)	−0.192	0.154	(0.0009)	7.100e+01
155.70	−0.091	(0.0120)	−0.121	0.059	(0.0021)	−0.106	0.050	(0.0005)	−0.052	0.059	(0.0006)	−0.180	0.149	(0.0007)	9.564e+01
158.90	−0.082	(0.0113)	−0.111	0.056	(0.0019)	−0.096	0.047	(0.0004)	−0.045	0.056	(0.0005)	−0.167	0.141	(0.0006)	1.389e+02
...															
600.70	0.398	(0.0153)	0.408	−0.021	(0.0144)	0.403	−0.018	(0.0138)	0.383	−0.021	(0.0138)	0.430	−0.053	(0.0137)	2.704e+06
603.30	0.395	(0.0154)	0.407	−0.023	(0.0145)	0.401	−0.019	(0.0139)	0.380	−0.023	(0.0139)	0.429	−0.057	(0.0135)	2.715e+06
606.70	0.392	(0.0158)	0.405	−0.025	(0.0147)	0.399	−0.021	(0.0140)	0.376	−0.025	(0.0140)	0.430	−0.063	(0.0137)	2.726e+06
...															
11691.20	0.028	(0.0200)	0.070	−0.084	(0.0076)	0.049	−0.070	(0.0054)	−0.028	−0.084	(0.0055)	0.155	−0.211	(0.0045)	1.332e+03
12153.40	0.026	(0.0208)	0.069	−0.087	(0.0077)	0.048	−0.073	(0.0054)	−0.032	−0.087	(0.0056)	0.157	−0.218	(0.0046)	1.146e+03
12432.30	0.024	(0.0209)	0.069	−0.089	(0.0077)	0.046	−0.074	(0.0053)	−0.035	−0.089	(0.0054)	0.158	−0.222	(0.0046)	1.050e+03

Table 2. Sample of the table with the passband-specific limb-darkening coefficients. We calculated the coefficients for the Strömgren *uvby* filters and for the Johnson-Morgan *UBVRI* filters. B2 and B3 are the response function of the *B* filter with and without the earth's atmosphere transmission function, respectively (see Buser 1978)

Model 5, $T_{\rm eff}$ = 3697 K, $F_{\rm rel}$ = 0.150, ν = 0.06943, $T_{\rm h}$ = 7000 K

	Linear Law		Quadratic Law			Cubic Law			Logarithmic Law			Square Root Law		
filter	x	Q	x	y	Q	x	y	Q	x	y	Q	x	y	Q
u	0.320	(0.0894)	0.575	−0.511	(0.0365)	0.448	−0.426	(0.0301)	−0.021	−0.511	(0.0296)	1.087	−1.279	(0.0259)
v	0.380	(0.0624)	0.518	−0.275	(0.0253)	0.449	−0.229	(0.0183)	0.197	−0.275	(0.0185)	0.793	−0.688	(0.0149)
b	0.317	(0.0563)	0.445	−0.256	(0.0250)	0.381	−0.214	(0.0188)	0.146	−0.256	(0.0189)	0.701	−0.641	(0.0156)
y	0.299	(0.0449)	0.396	−0.193	(0.0212)	0.348	−0.161	(0.0163)	0.171	−0.193	(0.0164)	0.589	−0.482	(0.0139)
U	0.265	(0.0887)	0.514	−0.498	(0.0356)	0.389	−0.415	(0.0287)	−0.068	−0.498	(0.0283)	1.012	−1.246	(0.0244)
$B2$	0.323	(0.0567)	0.456	−0.266	(0.0251)	0.389	−0.222	(0.0190)	0.145	−0.266	(0.0190)	0.722	−0.666	(0.0158)
$B3$	0.326	(0.0581)	0.460	−0.268	(0.0245)	0.393	−0.223	(0.0181)	0.147	−0.268	(0.0182)	0.728	−0.671	(0.0148)
V	0.307	(0.0410)	0.390	−0.166	(0.0201)	0.349	−0.139	(0.0159)	0.196	−0.166	(0.0159)	0.557	−0.416	(0.0138)
R	0.354	(0.0217)	0.383	−0.058	(0.0161)	0.368	−0.048	(0.0146)	0.315	−0.058	(0.0146)	0.441	−0.145	(0.0139)
I	0.291	(0.0195)	0.316	−0.049	(0.0147)	0.303	−0.041	(0.0135)	0.258	−0.049	(0.0135)	0.365	−0.124	(0.0129)

Table 4. Linear and quadratic limb-darkening coefficients for non-illuminated models. Our calculations were made with $T_{\rm eff}$ = 6700 K while Claret & Giménez (1990) used $T_{\rm eff}$ = 6730 K, both with $\log g = 4.5$. For the quadratic law, at the top is the linear coefficient and below it the non-linear one. Here CG stands for the results by Claret & Giménez (1990), CG′ and VH′ correspond to our calculations using the methods by CG and VH and, respectively, and, finally, VH correspond to the values by Van Hamme (1993), bi-linearly interpolated for $T_{\rm eff}$ = 6700 K and $\log g = 4.5$

band	Linear				Quadratic		
	CG	CG′	VH′	VH	CG	CG′	VH′
u	0.78	0.76	0.70	0.64	0.65	0.65	0.63
					0.14	0.15	0.14
v	0.79	0.77	0.72	0.66	0.68	0.67	0.66
					0.13	0.13	0.14
b	0.74	0.72	0.69	0.60	0.63	0.63	0.62
					0.13	0.13	0.13
y	0.65	0.63	0.59	0.51	0.51	0.52	0.52
					0.16	0.16	0.15
U	0.79	0.76	0.70	0.64	0.66	0.64	0.61
					0.15	0.16	0.17
B	0.76	0.74	0.70	0.62	0.64	0.64	0.63
					0.13	0.13	0.13
V	0.63	0.63	0.60	0.51	0.48	0.51	0.52
					0.17	0.15	0.15

a better fit. That was an expected result as CG does not require any constraint to the fitting, the coefficients being calculated directly by applying the least squares method to the integrated intensities obtained with the atmosphere model. As VH applies the constraint of total flux conservation the fittings are often worse, but always physically correct. We observe that the worse the fitting with VH method the more the CG coefficients fail in conserving the total flux ($|(F'/F_{\rm m})| > 1$).

As already noticed by Claret & Giménez (1990), the limb-darkening coefficients of an illuminated atmosphere strongly depend on many parameters, making it difficult to find a simple function that describes the effect. We propose, for the passband specific coefficients, a solution to easily account for the effect in LC synthesis programs, parametrizing with polynomials the results obtained. In Eq. (8) K represents x_λ or y_λ of Eqs. (1) to (4) for the different limb-darkening laws:

$$K = a_0(t) + \sum_{n=1}^{3} a_n(t,\nu,t_{\rm h}) F^n \tag{8}$$

$$a_n = \sum_{m=0}^{3} b_{nm}(\nu,t_{\rm h}) t^m \tag{9}$$

Table 3. Sample of the table with the polynomial adjusted coefficients for the Strömgren u filter. See Eqs. (2), (8)-(11)

	a0 - quadratic law											
	b00			b01			b02			b03		
	d00l0	d00l1	d00l2	d01l0	d01l1	d01l2	d02l0	d02l1	d02l2	d03l0	d03l1	d03l2
c0m0_L	10.93	−7.248	0.3957	−7.797	4.569	−0.2498	1.888	−0.9371	0.05130	−0.1432	0.06236	−0.003416
c0m1_L	−416.9	82.57	−4.093	254.7	−50.65	2.521	−50.50	10.09	−0.5041	3.259	−0.6536	0.03277
c0m2_L	891.2	−172.7	8.373	−541.8	105.4	−5.128	106.9	−20.86	1.019	−6.861	1.344	−0.06590
c0m3_L	−520.8	99.85	−4.792	315.9	−60.76	2.927	−62.15	12.00	−0.5800	3.982	−0.7710	0.03740
c0m0_NL	−34.82	13.11	−0.7404	25.86	−8.808	0.4929	−6.056	1.909	−0.1058	0.4440	−0.1320	0.007240
c0m1_NL	354.2	−85.30	4.891	−242.4	57.77	−3.283	53.71	−12.67	0.7140	−3.806	0.8899	−0.04969
c0m2_NL	−699.0	168.3	−9.626	475.2	−113.2	6.424	−104.9	24.75	−1.393	7.432	−1.737	0.09703
c0m3_NL	408.7	−98.24	5.601	−275.7	65.61	−3.714	60.54	−14.27	0.8019	−4.274	0.9982	−0.05570

	a1 - quadratic law											
	b10			b11			b12			b13		
	d10l0	d10l1	d10l2	d11l0	d11l1	d11l2	d12l0	d12l1	d12l2	d13l0	d13l1	d13l2
c1m0_L	−10.58	8.848	−0.5346	4.765	−5.034	0.3140	−1.547	1.077	−0.06571	0.1549	−0.07678	0.004508
c1m1_L	371.0	−85.77	4.585	−231.9	53.70	−2.906	50.95	−11.61	0.6254	−3.682	0.8205	−0.04368
c1m2_L	−700.3	154.4	−8.303	469.9	−102.7	5.530	−106.3	22.91	−1.222	7.773	−1.647	0.08689
c1m3_L	403.4	−84.34	4.455	−278.3	57.94	−3.059	63.51	−13.12	0.6877	−4.649	0.9513	−0.04941
c1m0_NL	−822.9	165.6	−9.820	472.6	−98.62	5.918	−88.36	19.05	−1.161	5.385	−1.198	0.07424
c1m1_NL	6542.	−1373.	73.18	−3589.	769.6	−41.78	642.6	−140.8	7.799	−37.59	8.431	−0.4767
c1m2_NL	−1.102e+04	2297.	−120.9	5963.	−1269.	68.04	−1051.	228.5	−12.50	60.32	−13.43	0.7516
c1m3_NL	5652.	−1172.	61.22	−3033.	641.4	−34.15	528.2	−114.1	6.209	−29.93	6.625	−0.3688

	a2 - quadratic law											
	b20			b21			b22			b23		
	d20l0	d20l1	d20l2	d21l0	d21l1	d21l2	d22l0	d22l1	d22l2	d23l0	d23l1	d23l2
c2m0_L	88.11	−19.03	0.9265	−49.12	10.90	−0.5373	10.17	−2.244	0.1103	−0.7167	0.1547	−0.007510
c2m1_L	−688.2	135.5	−6.444	415.8	−83.28	4.019	−88.61	17.84	−0.8650	6.285	−1.262	0.06110
c2m2_L	1032.	−202.5	9.791	−674.3	134.4	−6.580	152.2	−30.48	1.496	−11.21	2.243	−0.1098
c2m3_L	−511.1	96.06	−4.551	352.7	−67.97	3.276	−82.33	16.06	−0.7789	6.178	−1.212	0.05883
c2m0_NL	1247.	−237.2	12.57	−649.2	126.0	−6.796	109.8	−21.75	1.199	−6.061	1.226	−0.06922
c2m1_NL	−9325.	1806.	−89.12	4747.	−928.4	46.37	−779.6	154.1	−7.806	41.49	−8.294	0.4270
c2m2_NL	1.595e+04	−3051.	147.8	−7998.	1538.	−75.21	1285.	−248.5	12.28	−66.37	12.91	−0.6460
c2m3_NL	−8268.	1566.	−74.97	4099.	−778.6	37.54	−648.1	123.3	−5.992	32.75	−6.234	0.3057

	a3 - quadratic law											
	b30			b31			b32			b33		
	d30l0	d30l1	d30l2	d31l0	d31l1	d31l2	d32l0	d32l1	d32l2	d33l0	d33l1	d33l2
c3m0_L	−139.2	26.62	−1.250	77.80	−15.04	0.7118	−14.43	2.818	−0.1343	0.8846	−0.1742	0.008362
c3m1_L	825.5	−159.8	7.534	−476.3	93.27	−4.439	91.39	−18.10	0.8690	−5.795	1.160	−0.05615
c3m2_L	−1339.	261.1	−12.39	793.1	−156.5	7.499	−156.0	31.15	−1.505	10.12	−2.042	0.09950
c3m3_L	693.2	−134.1	6.334	−418.1	81.99	−3.912	83.57	−16.60	0.7991	−5.492	1.104	−0.05361
c3m0_NL	−1003.	190.1	−9.209	546.8	−104.4	5.076	−98.27	18.88	−0.9212	5.820	−1.124	0.05502
c3m1_NL	6296.	−1191.	56.17	−3357.	636.8	−30.09	586.9	−111.6	5.278	−33.72	6.420	−0.3039
c3m2_NL	−1.065e+04	1995.	−93.00	5615.	−1052.	49.05	−967.2	181.2	−8.435	54.64	−10.22	0.4749
c3m3_NL	5512.	−1026.	47.43	−2884.	535.9	−24.75	491.9	−91.20	4.198	−27.48	5.078	−0.2327

$$b_{nm} = \sum_{l=0}^{3} c_{nml}(t_{\rm h})\nu^l \tag{10}$$

$$\text{and} \quad c_{nml} = \sum_{k=0}^{2} d_{nmlk} t_{\rm h}^k, \tag{11}$$

with $F = F_{\rm rel}$, $t = T_{\rm eff}\ 10^{-3}$ and $t_h = T_{\rm h}\ 10^{-3}$.

In Table 3 we give a sample of the adjusted polynomial coefficients obtained from the data in Table 2 and in Fig. 7 we show the adjusted polynomial surface for one chosen model. In the case of a non-linear law, we give in Table 3 the linear adjusted polynomial coefficients (cnm0_L, cnm1_L, cnm2_L, cnm3_L) followed by the non-linear ones (cnm0_NL, cnm1_NL, cnm2_NL, cnm3_NL). A complete version of Table 3 with the adjusted polynomial coefficients for the 9 chosen photometric filters (see Sect. 2.1) can be accessed in electronic form at the CDS.

2.3. A control

It is important to compare limb-darkening coefficients calculated from different atmosphere models and computational methods. Claret & Giménez (1990) have also used the UMA code in their calculations, and our results can be easily compared. In Table 4 we show the coefficients for the linear and quadratic laws for non-illuminated atmospheres, obtained by Claret & Giménez (1990), Van Hamme (1993, only for the linear law, because VH did not include the quadratic law in his study) and by us, using

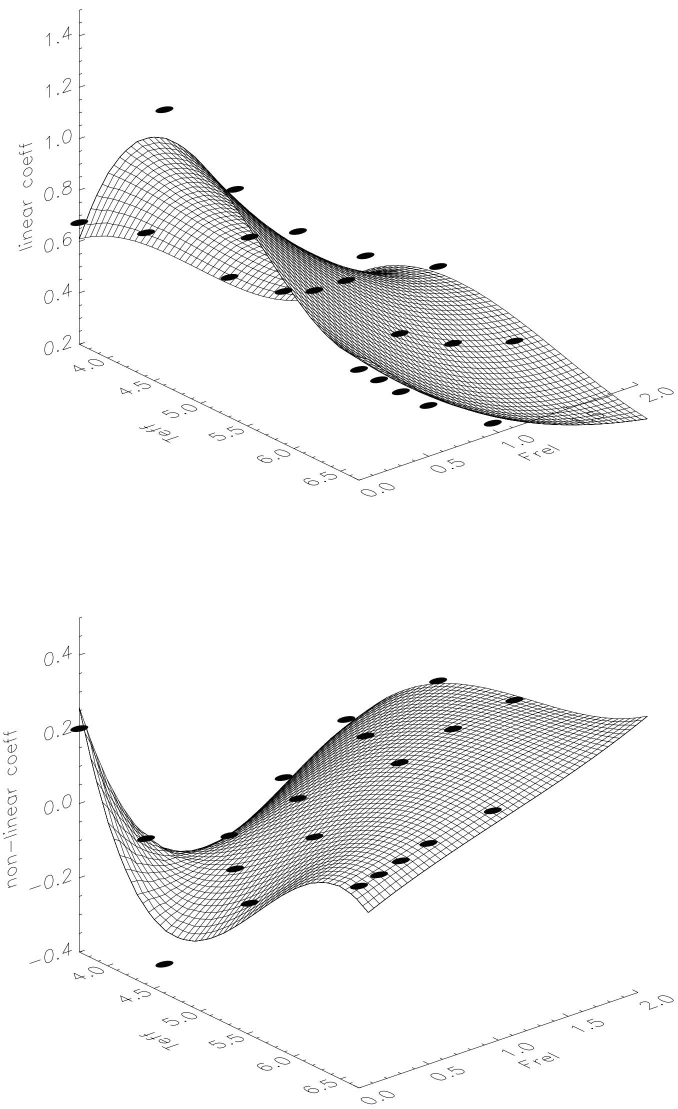

Fig. 7. Polynomial fittings to the quadratic law coefficients. Strömgren u filter, $\nu = 0.97$, $T_{\rm h} = 7000$ K. The x axis corresponds to $F_{\rm rel}$ and the y to $T_{\rm eff}$. The theoretical grid was calculated using the coefficients in Table 3

both VH and CG methods. As can be noticed, our values are very similar when using the same method but show a little expected difference with those determined by the VH method. Actually, Díaz-Cordovés & Giménez (1992) had already found that differences between tabulations by different works are normally due to the computational method rather than to the adopted model atmospheres themselves. This makes us confident that our calculations are correct for the non-illuminated case and that our results probably would be similar if we used another atmosphere model, instead of UMA. Note that the calculations with VH's method yield values somewhat systematically smaller for the linear law and that the limb-darkening coefficients given in VH (using ATLAS) are the smallest in each passband.

This control in the case of illuminated atmospheres is more problematic. To the best of our knowledge only CG worked on the effect of external irradiation on the limb-darkening coefficients using numerical atmosphere models (UMA in that case), but their results are only presented in a qualitative way, making impossible a numerical comparison. We do not know of any other work on the effect of external illumination on the limb-darkening coefficients with models other than UMA.

However, if the theoretical approximations used are similar to those used in this work we expect that the results will be similar to ours, irrespective of the atmosphere model used in the study. And we are confident that the results derived here are better, even used as a first order approximation, than the use of constant limb-darkening coefficients overall in the analysis of eclipsing binary LC.

3. Discussion

Until now the light curve synthesis programs have mostly used non illuminated limb-darkening tables to evaluate the center to limb brightness variation. In fact, the LC models use non-illuminated atmosphere results even to calculate the so-called "reflection effect", what is not strictly correct since the spectrum of an illuminated atmosphere is different from that of a standard one (Vaz & Nordlund 1992). The effect of the external irradiation on the coefficients themselves is ignored even when it is recognized that the limb-darkening coefficients should vary due to variations of the local effective temperature and surface gravity across the surface of a tidally distorted binary star, as in Van Hamme & Wilson (1994), who calculated position-dependent limb-darkening on computed light curves.

However, it is clear from Fig. 7 that the limb-darkening coefficients vary not only with the atmosphere's effective temperature, but show a strong dependence on the characteristics of the external illumination (the incidence angle, the amount of infalling flux, the spectrum of the infalling energy or, in other words, the temperature of the illuminating star), also. This dependence is well represented by the approximating Eqs. (8) to (11) for each of the 9 different limb-darkening laws reviewed in the present work. Even though the effect of the limb-darkening coefficients on the light curve analysis is small, it is large enough to allow attempts of empiric determination of the coefficients from precise observed light curves (as for DM Vir, Andersen et al. 1984), while the adoption of incorrect coefficients may affect systematically the determination of the orbital inclination, having also influence on the determination of other parameters (Popper 1984).

As the orbital configuration is fully known during the analysis of an eclipsing binary LC, Eqs. (8) to (11) can be used to calculate the limb-darkening coefficients which are position-dependent on the surface of the components, taking into account not only the variation of $T_{\rm eff}$ and $\log g$, but also the effect of the external irradiation. The parametrization attained is very convenient for this purpose, which can be implemented in the existing computer models for eclipsing binary light curve synthesis. This extra calculation will represent a relatively easy task for the modern CPU's and certainly will improve the quality and reliability of the determination of the other parameters as, for instance, the orbital inclination. In fact, the treatment of both the limb-darkening and the gravity brightening, with respect to the effect of the mutual illumination, should be done consistently, and this is our goal. In Alencar et al. (1998) we present the continuation of the work on the gravity brightening exponent (Alencar & Vaz 1997) extended to illuminated atmospheres. The implementation of these results in the WD model (Wilson & Devinney 1921; Wilson 1979; Vaz et al. 1995) is in progress and its application to real systems will be published elsewhere. While this is not the complete integration of atmosphere model calculation with the light curve synthesis programs, yet, a desirable feature of the next generation of the LC programs, this is a significant step towards the improvement of these models with respect to the proximity effects on the theoretical light curves.

4. Conclusions

We determined monochromatic, bolometric and passband specific limb-darkening coefficients for illuminated atmospheres. Our results show that illuminated coefficients are significantly different from the non-illuminated ones and that illuminated atmospheres may present limb brightening instead of darkening, depending on the set of parameters chosen. We tested two different methods (Claret & Giménez 1990 and Van Hamme 1993) to calculate the coefficients and showed that in the illuminated case the method proposed by Van Hamme is recommended in order to obtain coefficients that preserve the total emergent flux from the atmosphere. For the passband specific coefficients we also present our results in a polynomial form, which is convenient for implementation of the use of these "illuminated" coefficients in the analysis of eclipsing binary light curves.

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Appendix: Acronyms

Following a suggestion of the referee, we present here the definition of the many acronyms used in this paper.

$T_{\rm eff}$: effective temperature of the atmosphere to be illuminated
$T_{\rm h}$: heating star's effective temperature
γ: line of sight angle (see Fig. 1)
μ: $\cos\gamma$
θ: incidence angle (see Fig. 1)
ν: $\cos\theta$
λ: wavelength
$I_\lambda(\mu)$: beam intensity at the wavelength λ, direction μ
$R_\lambda(\mu)$: limb-darkening law
x_λ: linear limb-darkening coefficient
y_λ: non-linear limb-darkening coefficient
$r_{\rm h}$: the apparent radius of the illuminating star (see Fig. 1)
g: local surface gravity acceleration
α: the mixing length parameter
β: the gravity brightening exponent
Q: the quality factor of the limb-darkening law adjustment
$F(\lambda)$: the monochromatic flux
$F_{{\rm rel},\nu}$: bolometric incident flux relative to the one emitted by the illuminated star at the illuminated point of its surface
F': total emergent flux obtained by integrating the intensities calculated from the adjusted limb-darkening coefficients
$F_{\rm m}$: total flux obtained from the model intensities.

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