Astron. Astrophys. 326, 257–262 (1997)

ASTRONOMY
AND
ASTROPHYSICS

The gravity-brightening effect and stellar atmospheres

I. Results for models with 3700 K $<$ T$_{\rm eff}$ $<$ 7000 K

S.H.P. Alencar and L.P.R. Vaz

Departamento de Física-ICEx-UFMG, Caixa Postal 702, 30.161-970 Belo Horizonte MG, Brazil

Received 8 August 1996 / Accepted 18 March 1997

Abstract. The gravity-brightening exponent (β) is studied using the UMA (Uppsala Model Atmosphere) code in non-illuminated convective grey (in the sense of "continuum only" opacity) and non-grey (line blanketed) atmospheres, for $3\,700\,{\rm K} < T_{\rm eff} < 7\,000\,{\rm K}$. The value β=0.32 proposed by Lucy was confirmed only for $T_{\rm eff} \sim 6\,500\,{\rm K}$. β depends upon $T_{\rm eff}$ (an analytical expression is given), being rather insensitive to variations of the mixing length parameter, of the stellar mass and to the use of grey or non-grey atmospheres. A comparison with empirical determinations and a preliminary test of the use of the proposed β values in eclipsing binary light curve analysis are presented. A dependence of β on external illumination ("reflection effect") for convective models is evidenced for the first time, the larger the amount of incident flux the larger the β exponent.

Key words: atmospheres – stars: binaries: close – stars: binaries: eclipsing – stars: fundamental parameters – stars: general

1. Introduction

The analysis of eclipsing binary (EB) light curves (LC) involves a large number of parameters. To avoid multiple solutions, as many parameters as possible are kept fixed either at the observed values (e.g. mass ratio) or at the theoretical ones (limb-darkening coefficients, bolometric albedos, gravity-brightening exponents). LC generation models are able to directly adjust such parameters, but this may introduce correlations that make the convergence to physically acceptable solutions difficult or impossible.

The limb-darkening effect on EBLC is moderate (Popper 1984). Its coefficients for different laws (e.g. Al-Naimiy 1978, Wade & Ruciński 1985, Van Hamme 1993, Claret et al. 1995, amongst other) are satisfactorily used in the EB models, but the Strömgren u theoretical values seem not to correctly reproduce the observed u LC of early systems (Vaz et al. 1995 and references therein).

Send offprint requests to: L.P.R. Vaz

The bolometric albedos affect the LC of all types of close EB (Vaz 1984, 1985). The albedos for convective grey (Vaz & Nordlund 1985) and non-grey (Nordlund & Vaz 1990) atmospheres have been modeled. In non-grey models no convenient expression for the study of EBLC was found, and each case should be treated individually.

1.1. Basic works on the gravity-brightening effect

Von Zeipel (1924) showed that the local emergent flux of a star with an atmosphere in hydrostatic and radiative equilibrium and distorted by rotation or tidal effects is proportional to the local gravity acceleration (β = 1.0):

$$\mathscr{F} \propto g^{\beta} \qquad (1)$$

Lucy (1967) found that Eq.(1) would hold for convective atmospheres if $\beta \simeq$0.32, a value obtained by coupling convective envelopes of models with different surface gravities at depths where the temperature gradient was adiabatic. He recognized that the entropy at the bottom of different models should be the same, in order to represent the same star: "inward integrations of the equations governing the atmospheric structure must always end up on the same adiabat" (Lucy 1967). Lucy made his calculations for the effective temperatures corresponding to stars with $1.00\,{\rm M}_\odot$ (6 000 K) and $1.25\,{\rm M}_\odot$ (6 700 K and 5 950 K), using atmosphere models by Baker & Temesváry (1966).

Anderson & Shu (1977) presented another formulation for convective atmospheres arguing that, for a contact star in hydrostatic equilibrium having a common convective envelope (the contact discontinuity model, Shu et al. 1976, Lubow & Shu 1977), the convective flux depends on the effective gravitational potential alone, and not on its gradient (i.e. the local acceleration of gravity). As for late-type stars $F_{\rm conv} \simeq F_{\rm total}$ and the photosphere must ultimately radiate $\sigma T^4_{\rm eff}$, this flux should be constant on equipotentials, with consequently β=0.

1.2. Empirical determinations and modern works

Rafert & Twigg (R&T, 1980) determined β by analysing a uniform sample of detached, semi-detached and contact EB with the WD (Wilson 1979) program. They found $\beta \approx 0.31$ for convective envelopes ($5\,400\,\mathrm{K} < T_{\mathrm{eff}} < 7\,100\,\mathrm{K}$) and $\beta \approx 0.96$ for the radiative ones ($T_{\mathrm{eff}} > 7\,900\,\mathrm{K}$). For $7\,100\,\mathrm{K} < T_{\mathrm{eff}} < 7\,900\,\mathrm{K}$ they found empirically $0.31 < \beta < 1$.

Hilditch (1981) found very small values of β (four contact EB), and that imposing β=0.32 yields a photometric mass ratio which is significantly different from the spectroscopic one, what does not happen with β=0. However, the obvious presence of star spots in all the 4 systems may render very difficult, and even spoil, attempts to determine temperature variations over the surfaces (Ruciński 1989).

Nakamura & Kitamura (1992 and references therein) did empirical studies on the β exponent (their distortion theory extended to a second order treatment, Kitamura & Nakamura, K&N 1983). For detached EB their values agree with those of von Zeipel (1924) and Lucy (1967), but for early-type both semi-detached (K&N 1987b, 1992) and contact EB (K&N 1988a,b), they obtained $\beta > 1.0$.

Sarna (1989), with a model of convective envelope for Roche lobe filling stars (W UMa-type), computed the distribution of effective temperatures and surface gravities for the stars and directly determined $\beta \approx$0.32.

Observation and theory thus generally agree with each other, with $\beta \approx$1.0 for radiative atmospheres and $\approx$0.32 for the convective ones, albeit exponents $>$1.0 were found by K&N. There is observational evidence for a transition between these two regimes (K&N and R&T), suggesting that β=0.32 is valid only for temperatures close to what Lucy (1967) used. "Whereas β_{rad}=1 is a solid, physically well understood upper limit (...), β_{conv}=0.32 is an approximate relation based on a small set of envelope models which used very crude physics" (Ruciński 1989). Further, there is only one empirical determination (Ruciński 1976) and no theoretical study for $T_{\mathrm{eff}} < 5\,400\,\mathrm{K}$ up to now, all this being the motivation for the present study.

2. The atmosphere model

We use the Uppsala Model Atmosphere (UMA, Gustafsson et al. 1975, Bell et al. 1976) code, in a version by Vaz & Nordlund (1985). The code is designed for cool stars ($T_{\mathrm{eff}} < 8\,000\,\mathrm{K}$), atmospheres in hydrostatic equilibrium using plane-parallel structure, local thermodynamic equilibrium and modeling convection through mixing length.

The effective temperature T_{eff}, the surface gravity g, the stellar mass M, the mixing length parameter l/H_P, and the chemical composition (fixed at solar abundance in this work) define a model. We use stellar models (Claret 1995, X=0.70, Z=0.02) to associate T_{eff} with M for undistorted stars at the ZAMS. We refer to models with and without line absorption as "non-grey" and "grey", respectively, including in the "grey" models, though, the continuum opacity variation with frequency (as in Nordlund & Vaz 1990). We study grey (i.e. "continuum only") and non-grey (line blanketed or ODF) atmospheres in convective equilibrium. The ODF tables limit the temperature range of this study to those corresponding to (ZAMS) stars with masses ranging from $0.6\,\mathrm{M}_{\odot}$ to $1.5\,\mathrm{M}_{\odot}$.

2.1. The method

Starting from a reference model ($\log g$=4.5) we adjust T_{eff} for models having another $\log g$ (distorted) until the adiabat at the bottom of the distorted model becomes equal to that of the reference one, i.e. these models must represent the same star by having the same entropy at the bottom. We then examine how the total flux varies with g, determining β by linear regression.

Taking the most distorted components of some high quality EB solutions we find $\Delta \log g$=0.28 for V Pup (detached, Andersen et al. 1983), 0.22 for LZ Cen (detached, Vaz et al. 1995) and 0.30 for RY Aqr (semi-detached, Helt 1987). Larger values for $\Delta \log g$ will certainly be found amongst the most deformed components of contact EB. In the beginning of this study we used equally spaced steps of 0.1 in $\log g$ over a large interval of $\Delta \log g$=1.5 to calculate β. Equation (1) proved to be valid to a very high degree all over the interval, with both the β exponents and the linear regression correlation coefficients practically equal no matter if we used $\Delta \log g$=1.5 or smaller intervals (e.g. 0.30). However, an uncertainty of $\pm$5 K in the effective temperatures of the models could affect the β values for small $\Delta \log g$ intervals. Then, we decided to use in our study of grey atmospheres (Sect. 2.2) an interval $\lesssim$5 times the one easily happening for the components of close EBS, in order to minimize the effect of uncertainties in T_{eff} on β and to have our studies valid even for very distorted stars. In Sect. 2.3 we give β for both the large and the small $\log g$ intervals, but adopt the more realistic $\Delta \log g$=0.30.

To have layers deep enough for the adiabatic regime to be fully established, we extended the temperature structure from $\log \tau$=$-$4.2 to +7.3, going as deep in the atmosphere as the program allowed. In these deep layers F_{rad} is in practice zero, but there is non-adiabaticity because of mixing. The opacity hence is of negligible importance, unless it becomes completely wrong, what does not happen: the continuum opacity is indeed quite reasonable at the temperatures reached. The equation of state is close to modern formulations, such as the one by Mihalas et al. (1988), and is reasonable as well (it lacks effects like Coulomb screening, but these should come into play only much further down). Other formulations for the equation of state exist now, driven by helioseismology (Guenter et al. 1996, Elliott 1996, Antia 1996), but the level of differences are not significant in the current context. Moreover, these differences are particularly small at the surface, where the traditional (ODF) model atmospheres are actually covering effects not included in these newer formulations. The thermodynamic variables are both internally and physically consistent, as they obey the thermodynamic relations in the code (i.e., they stem from the same thermodynamic potential) and are supported by reasonable values for the continuum opacity and by the use of a realistic equation of state. Besides, these deep models produce the same spectrum as mod-

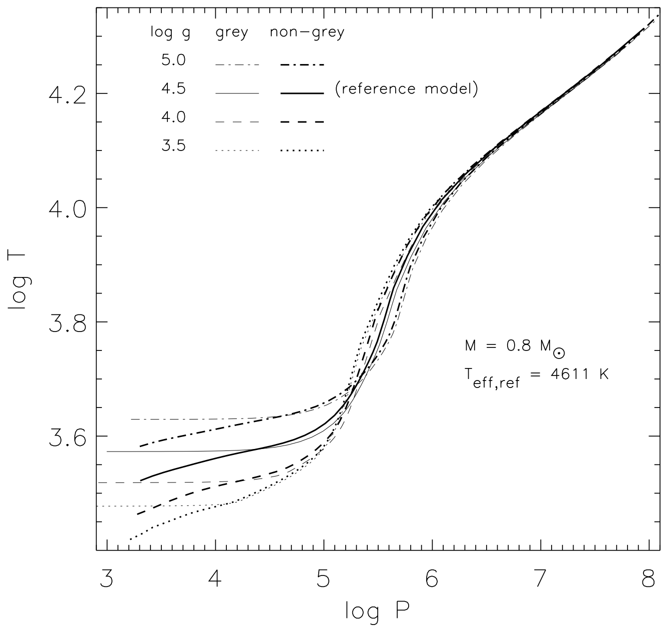

Fig. 1. $\log T$ vs. $\log P$ for convective grey (thin lines) and non-grey (thick lines) models with α = 1.5.

Table 1. Values of β for convective grey models. Models marked with an * are non-Main Sequence stars, used to investigate the influence of the mass and temperature on β. All models have solar chemical abundance.

$T_{\rm eff}$	M	$\alpha = l/H_P$				
(K)	($M_\odot$)	2.0	1.5	1.0	0.10	0.01
3 697	0.60	0.194	0.199	0.210	-	-
4 100	0.69	0.299	0.334	0.315	-	-
4 611	0.80	0.419	0.417	0.417	0.380	0.380
5 526	1.00	0.414	0.401	0.383	0.378	0.378
6 087	1.18	0.360	0.353	0.347	-	-
6 480	1.33	0.315	0.336	0.314	-	-
6 685	1.41	0.301	0.291	0.293	-	-
7 011	1.50	-	0.269	-	-	-
4 400*	1.65	-	0.389	-	-	-
4 800*	1.65	-	0.411	-	-	-
5 400*	1.65	-	0.407	-	-	-
6 000*	1.65	-	0.361	-	-	-
6 600*	1.65	-	0.302	-	-	-

Table 2. Coefficients for Eq. (2) in convective models.

	a_0	a_1	a_2	a_3
grey	−5.00784	27.9838	−46.9701	25.5398
non-grey	−5.61111	31.6225	−54.0000	29.7779

els (same parameters) having $\log\tau$ from -4.2 to $\sim$1.0. We are, therefore, still generating physically valid models.

The plane-parallel approximation means that we generate models for small regions over the distorted atmosphere (in terms of the geometrical depth our deep models have less than 0.6% of the star radius). Spherical symmetry models, apparently a better approach, would connect $\log g$ with M and R, and then have difficulties in generating a distorted star, where $\log g$ is larger precisely where the curvature radius is larger (but R is smaller). Both approximations, however, should lead to similar results for β with our method, as we compare models with the same approximation (in a way, β is a "differential" effect).

2.2. Convective grey (continuum only) atmospheres

Some models obtained with this method are shown in Fig. 1. The relation between $\log T$ and $\log P$ is the same at the bottom of the atmosphere in all the models that must represent different parts of the same distorted star.

We calculated models varying the values of $\alpha\,(=l/H_P)$, which is a measure of the degree of efficiency in the convective energy transport. However, β did not show a strong dependence on α, as shown in Table 1 and Fig. 2a. This was also noted by Sarna (1989). It can be seen, also, that the change in β with α is dependent on the model effective temperature. For some models ($0.8\,M_\odot$, $1.0\,M_\odot$) we calculated β for very small α values (0.1, 0.01). The resulting values show that β does not change significantly from its value for a normal atmosphere (α=1.5).

The last lines of Table 1 list tests made with non-main sequence stars (i.e. with a $T_{\rm eff}$ very different from the one a normal star with the same mass should have on this phase). The cold $1.65\,M_\odot$ model with $T_{\rm eff}$=4800K does correspond to a real Pre-Main Sequence (PMS) star (the secondary of TY CrA, Casey et al. 1993, 1995, 1997, see Sect. 3.2), while the other tests correspond to some different evolutionary PMS phases of that same star. These non-main sequence models show, however, that β depends mostly on $T_{\rm eff}$ for normal (i.e. non-illuminated, see Sect. 3.1) atmosphere models. Fig. 2a shows our theoretical values of β vs. $T_{\rm eff}$ listed in Table 1 and the third order polynomial adjusted with all the β values for grey atmospheres, given by the equation:

$$\beta = \sum_{i=0}^{3} a_i t_e^i \qquad (2)$$

with $t_e = T_{\rm eff} \times 10^{-4}$ and the values of a_i given in Table 2.

2.3. Convective non-grey (line blanketed) atmospheres

In Fig. 1 we have models calculated for convective non-grey atmospheres: the non-grey models differ from the grey ones being colder in the upper layers, and slightly hotter in the deep ones. For $T_{\rm eff}$>6 400 K we were forced to diminish the extension in $\log\tau$ in order to be able to calculate for the same range of effective temperatures used for the grey models, due to limitations of the ODF files.

β turned out to be very close to those for grey models, showing that the phenomenon is more related to the constitutive equa-

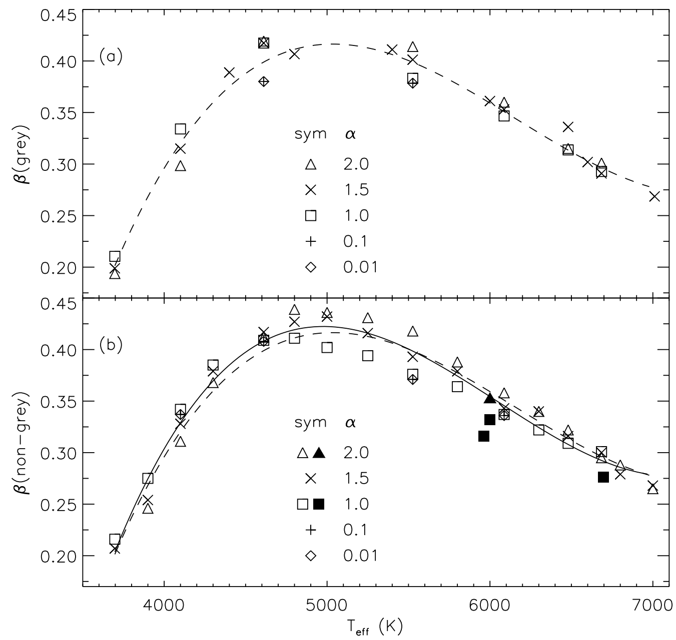

Fig. 2a and b. β vs. $T_{\rm eff}$ for convective (**a**) grey and (**b**) non-grey models (filled symbols are Lucy calculations for a similar chemical composition). The third order polynomials are shown.

Table 3. Values of β for convective non-grey models. The values for α=1.5* were calculated using $\Delta\log g$=1.5, while all other used $\Delta\log g$=0.3. In all cases the reference model has $\log g$=4.5.

$T_{\rm eff}$	M	$\alpha = l/H_P$					
(K)	($M_\odot$)	2.0	1.5*	1.5	1.0	0.1	0.01
3 697	0.60	-	0.205	0.207	0.216	-	-
3 900	0.66	0.246	0.253	0.254	0.275	-	-
4 100	0.69	0.311	0.322	0.328	0.342	0.337	0.337
4 300	0.75	0.368	0.369	0.379	0.385	-	-
4 611	0.80	0.412	0.413	0.417	0.409	0.409	0.408
4 800	0.83	0.439	0.430	0.427	0.411	-	-
5 000	0.87	0.436	0.432	0.432	0.402	-	-
5 250	0.93	0.431	0.414	0.416	0.394	-	-
5 526	1.00	0.418	0.396	0.393	0.376	0.372	0.371
5 800	1.09	0.388	0.375	0.379	0.364	-	-
6 087	1.18	0.358	0.349	0.343	0.337	0.336	0.336
6 300	1.26	0.340	0.336	0.340	0.322	-	-
6 480	1.33	0.322	0.321	0.316	0.309	-	-
6 685	1.41	0.295	0.298	0.300	0.301	-	-
6 800	1.44	0.288	0.282	0.279	-	-	-
7 000	1.50	0.265	0.272	0.268	-	-	-

tions than to the ODF table used or to the value of the mixing length parameter, α.

The weak dependence of β on α is confirmed for non-grey models (Table 3), although some trends become evident: the maximum of β shifts slightly, but systematically, in $T_{\rm eff}$ with α (Fig. 2b). However, the influence of α on β (same $T_{\rm eff}$) is small if compared with the normal errors involved in the empirical determinations of β (see Sect. 3.1). Then, we used all values of α (calculated with $\Delta\log g$=0.3) in fitting the third order polynomial to the non-grey β (Table 2, Fig. 2b) which should be the best approximation for real stars.

The β values calculated by Lucy (1967) are shown in Fig. 2b, also. Although the procedure used to compute β was the same, our opacity tables and atmosphere model were different from those he used. Even then, our results are in very close agreement: the mean of the 16 values of Table 3 calculated with $\Delta\log g$=0.3 in the interval 6 000 K $\leq T_{\rm eff} \leq$ 6 800 K is β = 0.320 $\pm$ 0.006.

3. Discussion

Our results for the β exponent in continuum only and line blanketed convective atmospheres are very similar to each other. The dependence of β for these models is mostly on the effective temperature, being very weak, albeit systematic, its dependence on the mixing length parameter (i.e. on the efficiency of convection in the energy transport).

For lower $T_{\rm eff}$, convection becomes very efficient due to the higher density caused by the lower opacity. The entropy jump at the surface becomes small and its changes even smaller (the convective zone reaches layers closer to the surface), yielding small β. For increasingly higher $T_{\rm eff}$, the top convective layers start deeper and deeper, increasing the entropy jump and its changes at the surface and, consequently, β. At high $T_{\rm eff}$, convection loses importance altogether, and the effect weakens again, although the adiabat still is a constraint at the bottom. In between these limits β reaches a maximum at $\sim$5 000 K.

Equation (1) was valid to a high degree in all the models. The β values were determined by linear regression with correlation coefficients very close to 1.0: the smallest value was r^2=0.994 for some models with α=2.0. The correlation coefficients for the β values calculated by using either $\Delta\log g$=0.30 or 1.5 (see Table 3) were essentially equal.

3.1. Comparison with empirical determinations

Figure 3 shows our results for β and empirical determinations found in the literature for the exponent. The 1σ error bars are given for the β exponent only, due to lack of information on the $T_{\rm eff}$ errors in most of the works used. However, we may assume that these errors are at least $\pm$500 K, a too optimistic figure for works using UBV, but reasonable for those using $uvby$ or DDO photometry.

Lucy's (1967) β=0.32 is a reasonable approximation for the data in Fig. 3. Our theoretical result agrees with Lucy's value in the temperature range he used and matches most of the empirical values for detached systems in Fig. 3, with only one determination showing a large discrepancy. Although the values for semi-detached and contact systems show a larger scattering, the theoretical curve still is a good approximation. One reason for the large scattering of the empirical β values for the more

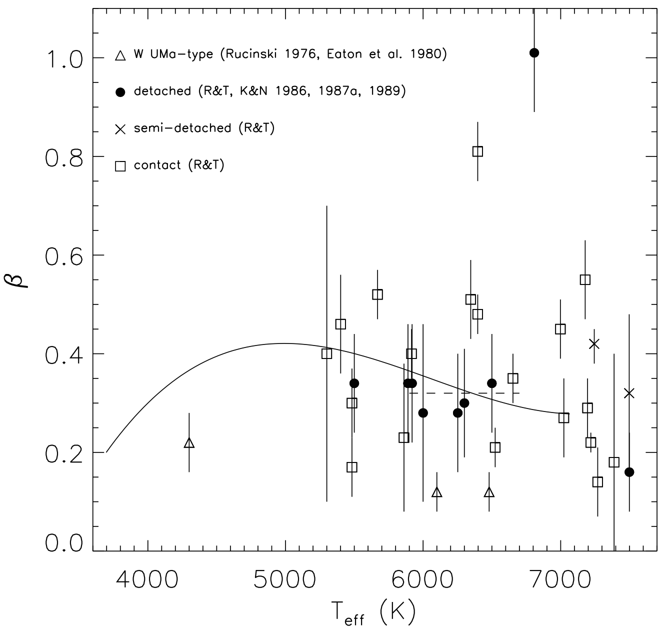

Fig. 3. The β theoretical curve and empirical results. The dashed line is Lucy's value in the temperature range he used.

deformed systems may be the large numerical correlation in the EB models between the exponent and the geometric parameters (inclination and star sizes) and, in some configurations, with the effective temperatures, too. Note that the above mentioned correlation has meaning exclusively for the empirical determinations of β; the theoretical values depend only on the atmosphere model parameters.

Contact and semi-detached EB should be better represented by illuminated atmospheres, since their components are closer to each other. Some calculations were made with an atmosphere grey model of $T_{\rm eff}$=4 611 K heated by sources with $T_{\rm h}$=4 500 K and 7 000 K, following Vaz & Nordlund (1985). The amount of incident flux may be changed through the value of $r_{\rm h}$, the ratio between the source star radius and the distance from its centre to the point at the surface of the reflecting star. The results (Table 4) show that the external heating increases the β values roughly in proportion to the amount of the incident flux. It would be then expected that contact and semi-detached systems would lie above our polynomial fit computed for non-illuminated atmospheres. The error estimations for β in Fig. 3 may be too optimistic, due to the correlations mentioned above and to the intrinsic difficulties in its empirical determinations. Better determinations of β are needed to confirm observationally the theoretically expected influence of external illumination[1].

The system CC Com (Ruciński 1976) has the only observational determination of β for $T_{\rm eff}$<5 400 K. W UMa (Eaton et al. 1980) has both components shown in Fig. 3 with $T_{\rm eff}$ by Hutchings & Hill (1973) and presents the lowest β value amongst all the systems. Our model cannot reproduce the β=0 predicted by Anderson & Shu (1977) for $T_{\rm eff}\lesssim$6 400 K. The reason may be the fact that our method is unable to detect any horizontal entropy gradients, which are possibly significant in contact systems, but not in the (semi) detached ones. Our results, then, must be taken with care if applyed to contact systems, but should be valid for the detached and semi-detached ones.

[1] Considering the □ and × symbols, there are more determinations above our curve (12, being 7 by more than 1σ) than below (7, 5 by more than 1σ) in Fig. 3.

Table 4. β for convective grey models with $T_{\rm eff}$=4 611 K illuminated by different stars in various degrees of approximation. The external flux incidence angle with the surface normal is 21°.5. No illumination is represented by $r_{\rm h}$=0.00.

$T_{\rm h}$	Apparent radius $r_{\rm h}$				
(K)	0.00	0.05	0.10	0.20	0.40
4500	0.417	0.422	0.427	0.439	0.494
7000	0.417	0.426	0.451	0.515	0.615

3.2. One first observational test

Equation (2), with the non-grey coefficients of Table 2, was implemented in the modified version of the WD model (Wilson & Devinney 1971, Wilson 1993) described by Vaz et al. (1995). The triple PMS eclipsing system TY CrA (Casey et al. 1993, 1995) has its eclipsing secondary component early in the PMS evolutionary phase, at the very end of the Hayashi track. The light curves were analysed (Casey et al. 1997) with the WD model using both Lucy's value and our approximation for β. For the secondary's effective temperature (4 800 K) our value is 0.41, about 28% larger than Lucy's value. The results obtained with our β were more consistent than when using β=0.32, yielding a (B8-9 ZAMS) primary star with absolute dimensions closer to what theoretically expected for a star in such phase. The reason was the increase of surface brightness in the secondary polar regions, requiring a lower inclination to reproduce the minimum depth and, consequently, larger stellar radii to reproduce the minima duration. Even though this result is encouraging, TY CrA is not, however, the most adequate system for such a test, as it is in a problematic evolutionary phase. We are searching for more favorable EBS for a definite observational test of the impact of the proposed theoretical values for β on EBLC analysis, whose results will be published elsewhere.

4. Conclusions

The gravity-brightening law, Eq. (1), is valid to a high degree for all models generated in this work with a modified version of the UMA code (plane-parallel atmosphere model in both hydrodynamic and local thermodynamic equilibrium). The β exponents for convective atmospheres were calculated theoretically by forcing the entropy at the bottom of the atmosphere to be equal in models that should represent the same star. Our model reproduces the mean result of Lucy (1967) for the convective regime.

We extended the result of Lucy (1967, for three values of $T_{\rm eff}$) to the range 3 700 K$<T_{\rm eff}<$7 000 K and found that β practically only depends on $T_{\rm eff}$, being rather insensitive to the value of the mixing length parameter, to the use of grey or non-grey atmospheres, and to the model total mass. A third order polynomial, $\beta(T_{\rm eff})$, was given as a convenient way to use the results in the analysis of EBLC.

The influence of the "reflection effect" on the gravity-brightening exponent for convective atmospheres is evidenced for the first time: the external illumination increases the values of β significantly, indicating that in close systems, where the reflection effect is prominent, the standard β values might be too low. A detailed analysis including illuminated models and an extension of the present work to higher effective temperatures are in progress.

Acknowledgements. Support by FAPEMIG, CNPq, FINEP, CAPES (Brazilian institutions). Fruitful discussions with Dr. Å. Nordlund and Dr. J.V. Clausen are gratefully acknowledged. The comments of an anonymous referee were fundamental to bring this work to its final form.

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