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Enhancement Factor on Tip-Enhanced Raman Spectroscopy

Dissertação apresentada ao Programa de Pós-Graduação em Física do Instituto de Ciências Exatas da Universidade Federal de Minas Gerais como requisito parcial para obtenção do título de Mestre em Ciências.

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ATA DA SESSÃO DE ARGUIÇÃO DA 643ª DISSERTAÇÃO DO PROGRAMA DE PÓS-GRADUAÇÃO FÍSICA DEFENDIDA POR AROLDO RIBEIRO LOPES NETO, orientado pelo professor Ado Jório de Vasconcelos e coorientado pelo professor Luiz Gustavo de Oliveira Lopes Cançado para obtenção do grau de **MESTRE EM FÍSICA**. Às 14:00 horas de vinte e nove de maio de 2020, por videoconferência, reuniu-se a Comissão Examinadora, composta pelos professores **Ado Jório de Vasconcelos** (Orientador - Departamento de Física/UFMG), **Luiz Gustavo de Oliveira Lopes Cançado** (Coorientador - Departamento de Física/UFMG), **Leonardo Teixeira Neves** (Departamento de Física/UFMG) e **Wagner Nunes Rodrigues** (Departamento de Física/UFMG) para dar cumprimento ao Artigo 37 do Regimento Geral da UFMG, submetendo o bacharel **AROLDO RIBEIRO LOPES NETO** à arguição de seu trabalho de dissertação, que recebeu o título de **"Enhancement Factor on Tip-Enhanced Raman Spectroscopy"**. O candidato fez uma exposição oral de seu trabalho durante aproximadamente 50 minutos. Após esta, os membros da comissão prosseguiram com a sua arguição e apresentaram seus pareceres individuais sobre o trabalho, concluindo pela aprovação do candidato.

Belo Horizonte, 29 de maio de 2020.

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Resumo

A espectroscopia Raman com aumento por ponta (do inglês *tip-enhanced Raman spectroscopy* - TERS) é uma técnica que combina o potencial da espectroscopia Raman de fornecer informações químicas sobre uma amostra com a capacidade da microscopia de varredura por sonda (do inglês *scanning probe microscopy* - SPM) de detalhar aspectos espaciais com resolução da ordem de nanômetros. A nanoantena é o dispositivo por trás do aumento de resolução do TERS, sendo geralmente feitas por um metal com propriedades plasmônicas e tendo o ápice com dimensões nanométricas. Existem várias técnicas diferentes para produzi-las, e algumas proporcionam aumentos de resolução maiores que outras. Porém, vários outros fatores não relacionados com a ponta também contribuem para esse aumento, e não há consenso sobre uma definição do fator de aumento intrínseco (f_e) fornecido pela nanoantena, que isolaria esses outros fatores. A complexidade reside na ausência de uma definição teórica adequada para esse fator e de um experimento capaz de fornecer resultados consistentes para medi-la. Esta dissertação propõe um protocolo capaz de medir este f_e. A demanda dessa definição vem da rápida evolução do TERS e das tecnologias da construção dessas antenas, e da necessidade de se determinar quais são os fatores modificáveis que afetam as propriedades de aumento das pontas. Primeiramente, nesta dissertação foram descritos os aspectos teóricos da curva de aproximação, e como esses parâmetros se comportam ao vários parâmetros da amostra e da ponta. Em seguida, foi discutido um dispositivo e um protocolo para medir o fator intrínseco f_e. Essa medida deve ser realizada em um material de referência feito que tenha um comportamento de espalhamento Raman de campo próximo bem descrito na literatura, integrado em um dispositivo com outras especificações que permitem caracterizar e otimizar o sistema de medição. Por fim, no capítulo de conclusão, duas possíveis aplicações para o protocolo são descritas.

Palavras-chave: Nanoantena, Fator de Aumento, Grafeno, Material de Referência, Tip-enhanced Raman Spectroscopy

Abstract

Tip-enhanced Raman spectroscopy (TERS) is a technique that combines the Raman spectroscopy capability of providing chemical information with a Scanning Probe Microscopy (SPM) potential to resolve spatially details in the order of nanometers. The specific device behind the TERS spatial resolution enhancement is the nanoantenna. The nanoantennas are usually made of metal, and their tip apex is of the order of only a few nanometers. Also, there are several different ways to produce nanoantennas that yield different spatial resolution. However, other factors not related to the nanoantenna also play a role in the enhancement, and there is up to date no agreement on what is the intrinsic enhancement factor (f_e) provided by the nanoantenna. The complexity resides both on the lack of a proper theoretical definition and of a reliable experimental procedure for measuring it. This dissertation aims to propose a protocol to measure the f_e. The demand comes from the rapid development of TERS and the engineering of this nanoantennas, and the necessity to establish the tailorable factors that affect the enhancement properties of these tips. First, the theoretical description of the tip-sample interaction is investigated, as well as how it behaves by varying the sample and tip parameters. Latter, a tool for measuring the intrinsic f_e and a protocol are discussed, which should be performed in a reference device made of a well-defined near-field Raman scattering material, integrated in a calibration device that that allows measuring all the parameters necessary to fully optimize and characterize the measuring system. In the conclusion chapter, two applications of the protocol are described.

Keywords: Nanoantenna, Enhancement Factor, Graphene, Reference Material, Tip-enhanced Raman Spectroscopy

Contents

1 Introduction

When a nanoantenna is positioned on the light scattering region of a sample, it might amplify the signal. The Tip-enhanced Raman Spectroscopy (TERS) is one of the techniques that make use of this phenomenon to intensify the usually low Raman signal by having a Scanning Probe Microscopy (SPM) system positioning this nanoantenna in the desired location. Thus, the scattered light on the area below the nanoantenna apex is enhanced providing both chemical and topographic information with a nanometric spatial resolution.

This dissertation is dedicated to understand the underlying phenomena of the Raman signal enhancement by the nanoantenna and to propose technological developments related to this knowledge. This Introduction chapter has a brief description of TERS in graphene, which is the prototype material utilized for measuring tip enhancement. It is followed by an explanation of the nanoantennas' role in the enhancement and a presentation of their fabrication methods. The following chapter, namely Review of the Enhancement Factor Measurements, is dedicated to review the problem of defining the enhancement factor of a nanoantenna, and to argue why they are problematic. The third chapter, entitled Theoretical Description of a Tip-approach Curve describes the theory to calculate the signal intensity in a TERS experiment, and the results are used to define the intrinsic enhancement factor f_e and to derive The Calibration Device and Measurement Protocol for measuring f_e. The Conclusion highlights two examples of protocol applications: measuring enhancement factor of a tip and accessing other 2D material properties, i.e. as the phonon spatial coherence length L_c.

1.1 Conventional Raman Spectroscopy versus TERS

TERS is able to provide chemical information with an improved spatial resolution when compared to conventional Raman spectroscopy [1–4]. In order to illustrate this, Fig.1 shows in (a) a conventional Raman image and (b) the TERS-image of the same carbon nanotube (CNT) serpentine, as measured in Ref. [5]. The conventional Raman hyperspectra resulted in a blurred image with a spatial resolution of $400\ \mu m$, the size of the laser spot. Then, the TERS was performed in the box region of 1(a): the result shown in 1(b) evidences a more defined CNT structure, revealing details that were not observable in the conventional Raman image. The spatial resolution was improved to $25\ nm$, which is roughly the size of the nanoantenna apex [5].

Given that TERS is a non-destructive and label-free procedure to access both chemical and topographic information with a high spatial resolution, several applications

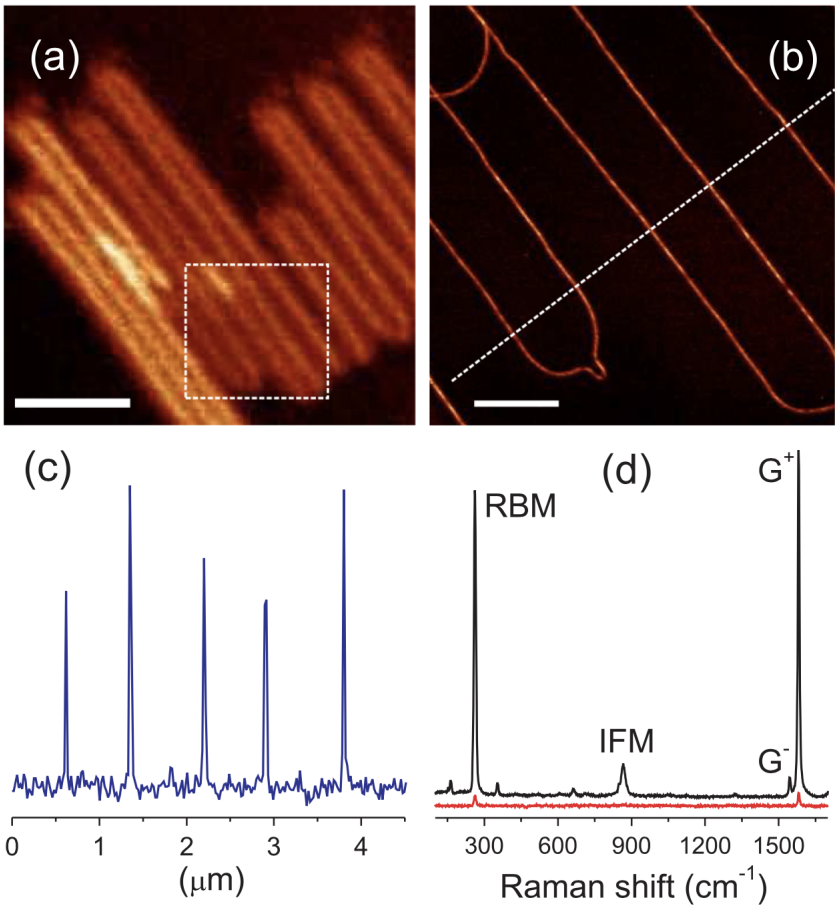

Figure 1 – Conventional Raman spectroscopy image versus TERS image.
(a) Conventional confocal Raman of a carbon nanotube serpentine. (b) TERS image of the boxed region in (a). Reproduced from Ref. [5].

have been reported: single-cell membranes components dynamics were measured [6–8]; single RNA-bases were labeled [9–11]; single virus particles were detected [12–14]; protein structural differences were observed [15–17]; strain on silicon devices and carbon-based devices were characterized [18–20].

1.2 The Nanoantenna

The key technological device behind the TERS effect is the nanoantenna. When this sharp metal tip is added to the SPM system and illuminated by a light source, it occurs the coupling of the photons with the free oscillation of the electrons on the metal surface, forming the surface plasmon-polariton (SPP) [21]. Due to the shape of the nanoantenna, the SPP concentrates in the vicinity of the nanoantenna's tip, strongly enhancing the electric field $|\mathbf{E}|$ that is now confined to a region proportional to the area below the nanoantenna's tip apex [22–24]. Also, due to the sharpness of the tip apex, the lightning rod effect plays a non-negligible role in the enhancement [25].

The polarization of the laser beam that excites the nanoantenna also contributes to the enhancement: The enhancement is 10^3 larger when the electric field has a strong component along the nanoantenna axes, and a smaller contribution from the perpendicular components. This configuration obtained when the a radially polarized laser beam is focused using a high numerical aperture (N.A.) lens [23, 26, 27].

Another important aspect of TERS comes from the fact that the nanoantenna is positioned only a few nanometers above the sample, working in the near-field regime. In this regime, the evanescent electric field emitted by the nanoantenna is now able to excite the sample, otherwise, if the nanoantenna is positioned more than a hundred nanometers from the sample, the magnitude of the evanescence electric field goes to zero and no longer excites the sample [23]. The other way around is also true, the nanoantenna is able to collect the near-field Raman scattered light from the sample and to transform the information contained in a propagating far-field via the oscillations of the SPP. Therefore, the technique is able to capture the interference of the light scattered by the phonons because the photons have a correlation length L_c in the order of tens of nanometers, accessible by the nanoantenna in the near-field regime [28–31].

1.2.1 Nanoantenna Fabrication

The efficiency of the TERS relies on having a nanoantenna with a powerful enhancement factor (EF), therefore, it is desirable from a fabrication standpoint to develop methods to produce tips with highest EF, and also in a cheap and reproducible fashion.

One fairly commonly used technique to produce these tips is the lamella-dropoff electrochemical etching method [32–35]. This method consists of diving 100 μm thick gold or silver wire in an etching solution and passing a DC current through the wire. The applied tension is then turned off when a drop in the current is detected, meaning that the immersed part of the wire has etched enough to form a tip. Images a) and b) of Fig.2 show the Scanning Electron Microscopy (SEM) image of one nanoantenna produced by this etching procedure.

Other techniques have been developed to increase the nanoantenna EF by exploring cavity-induced resonance conditions of the SPP, via localized surface plasmons resonance (LSPR). One of these consists in producing a groove near the apex of a gold etched tip by using a focused ion beam (FIB) setup [36]. In the bottom of Fig.2 there is a comparison of the Raman intensity enhancement provided by the same tip before the milling, on the left (c), and after the milling, on the right (d). By comparing the height of the blue lines before and after the milling, it is confirmed the positive effect of the LSPR on the enhancement of the signal.

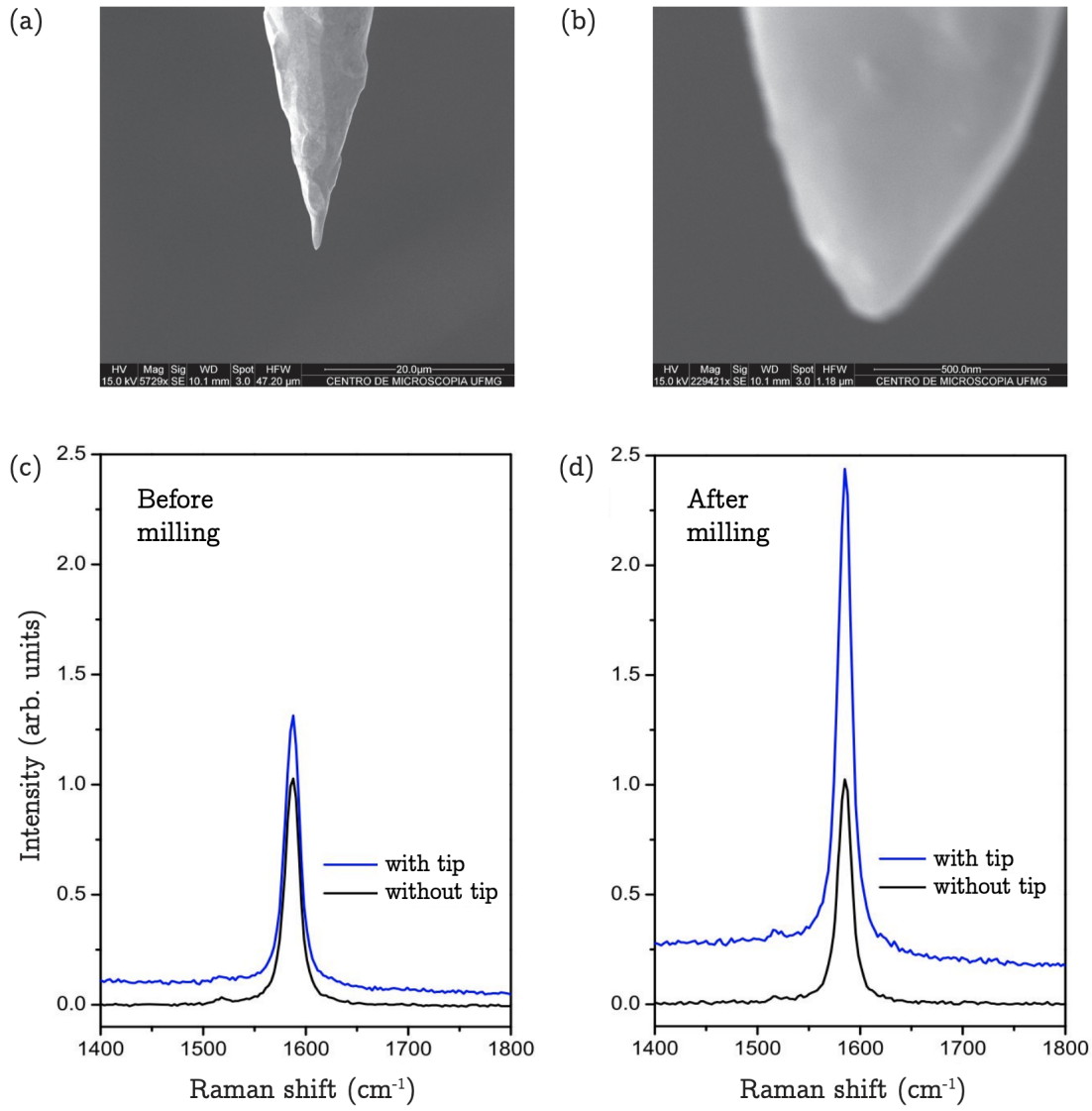

Figure 2 – Top: SEM image of an electrochemically etched tip. Bottom: Raman intensity enhancements before and after the FIB milling.

(a) and (b) figures show SEM images of a nanoantenna fabricated by the lamella-dropoff electrochemical etching method using HCl as the enchant. The b) figure is a zoom of the tip apex, estimated to be on the order of 30nm in diameter. The two bottom images were taken from Ref [36], where c) shows the enhancement of one tip made by a similar electrochemical process, and the d) shows a greater TERS enhancement by the same tip after FIB milling.

Many other techniques address the challenge to produce these nanoantennas. For instance, a standard AFM tip can be coated by chemical deposition of gold or silver [37–39], or fabricating nano-pyramids by taking advantage of the anisotropic etching of crystalline silicon and then depositing gold on the pyramid-shaped structure [40, 41]. It is also possible to tune the enhancement by creating FIB-induced LSPR conditions [42].

With all these possible methods to create nanoantennas, the question to answer is which has the greatest EF. The problem is that many variables that are not related to the nanoantenna can change the field enhancement. For instance, incident laser wavelength, specific metal, polarization and incident angle, alignment, etc, they are all instrumental factors that might influence in the enhancement [43–45]. Additionally, it is known that the substrate material and roughness influences the intensity [46, 47]. Furthermore, different authors calculate the EF in a different way, under different experimental condition and, consequently, it is not possible to infer which nanoantenna provides the greatest enhancement.

The main goal of this dissertation is to demonstrate a standardized way to measure the intrinsic enhancement factor of the nanoantenna, f_e [48]. This protocol makes it possible to compare two different nanoantennas by defining an intrinsic enhancement field factor f_e. In the next chapter, the literature will be revised on the current definitions on how to measure the EF.

2 Review of the Enhancement Factor Measurements

As stated in the end of the introductory chapter, several aspects of the configuration of a TERS experiment might change the observed enhancement. It is indeed a challenge to isolate the effect provided by the nanoantenna itself. In this chapter, it is presented a review on the most broadly used definition of $EnhancementFactor$, and the related problems are discussed.

2.1 The Enhancement Factor in the Literature

The simplest way to evaluate the nanoantenna in terms of its field enhancement EF is to measure the *contrast*, which is defined as the ratio of signal intensity measured with the nanoantenna and without it [40]:

$$\begin{aligned} contrast &= \frac{I_{near\,field}}{I_{far\,field}} \\ &= \frac{I_{with\,tip} - I_{without\,tip}}{I_{without\,tip}} \\ &= \frac{I_{with\,tip}}{I_{without\,tip}} - 1, \end{aligned} \tag{2.1}$$

where $I_{near\,field}$ is defined as the Raman peak intensity due only to the interaction of the nanoantenna with the scattering material, which is the signal measured while the tip is interacting with the sample, $I_{with\,tip}$, subtracted by the signal when the tip is not interacting with the sample ($I_{without\,tip}$). On the other hand, $I_{far\,field}$ is the intensity measured without the nanoantenna enhancement, which equals the intensity measured without the tip, $I_{without\,tip}$.

While the contrast is sufficient to determine the EF in the case of an ideal zero-dimensional scatterer, such as a single molecule, in the case of a sample dispersed in a surface or volume, the amount of material contributing to the far-field and near-field signals have to be evaluated. The EF then is defined as the product of the contrast and the effective illuminated area ratio between near- and far-field, namely the geometric factor GF, with

$$EF = contrast \times GF. \tag{2.2}$$

The GF evaluation depends on sample geometry, illumination pattern and tip properties. Considering a sample homogeneously distributed on a two-dimensional surface, we name

$A_{focus} = \pi r_{focus}^2$ as the illuminated area of the focused laser beam and $A_{tip} = \pi r_{tip}^2$ as the illuminated area of the tip apex on the scattering surface. GF can then be evaluated as

$$GF = \frac{A_{focus}}{A_{tip}} = \frac{r_{focus}^2}{r_{tip}^2}. \tag{2.3}$$

Although largely accepted due to its simplicity [22, 49–53], this "back-to-the-envelop" theory is problematic. Different samples in different setups, and even different scattering systems result in different contrast values for the same nanoantenna. Table 1 exemplifies how changes on r_{tip} and r_{focus} drastically vary the EF evaluation. Considering, without loss of generality, a measurement resulting in a *contrast* of 2.1, using a tip of radius $(15 \pm 5)nm$ and laser focus radius of $(200 \pm 40)nm$, the resulting EF using equation 2.2 varies from *134* to *1209*.

In Ref. [47] the authors reviewed several articles, each of them with a different TERS experiment configuration. All of them resulted in different *contrast* and EF values. The results are reproduced with adaptations in Fig.3. It is emphasized in the figure the different SPM based systems used in each work, where the dark green and dark red represents respectively the EF and the *contrast* of a TERS experiment having the SPM based on atomic force microscopy (AFM), and light green and light red represents the same properties but for SPM system based on scanning tunneling microscope (STM) [54].

Table 1 – EF values as a function of laser spot radius r_{focus} and tip radius r_{tip} calculated using Eqs. 2.2 and 2.3 for a $contrast = 2.10$.

		Laser spot radius (nm)				
		160	**180**	**200**	**220**	**240**
Tip Radius (nm)	**15.0**	261	331	408	494	588
	17.5	192	243	300	363	432
	20.0	147	186	230	278	331
	22.5	116	147	181	219	261
	25	94	119	147	178	211

When comparing different nanoantennas among the references in Fig.3, it is not possible to conclude exactly which one has the greatest intrinsic enhancement factor. For instance, in (m) Mehtani et al 2005 [55] it is reported a lower *contrast* than in (o) Neugebauer el al 2006 [56], but the EF values are in the same order of magnitude.

Consequently, different authors may define the enhancement factor slightly differently, depending on the application [51–53]. Also, there isn't an agreement in the TERS research community whether the EF is related to the experimental setup [22, 47, 49] or to the tip itself [50]. In order to have TERS as an useful nanomaterials' characterization technique, it is urgent to minimize the arbitrariness in the definition of enhancement factor of a nanoantenna, providing a protocol to compare different nanoantennas in terms of their enhancement factor.

The next chapters will be dedicated to derive the protocol to measure the intrinsic nanoantenna enhancement f_e. This protocol is based on measuring the TERS intensity as a function of the tip-sample distance Δz (named tip-approach procedure) of a graphene-based reference material. Subsequently, the experimental data is adjusted by the tip-approach curve $I_{\Gamma}^{TERS}(\Delta z)$, which represents the Raman scattering of intensity I of the phonons of symmetry Γ, given a tip-sample separation of Δz. In the following section, the theory behind the tip-approach curve will be presented.

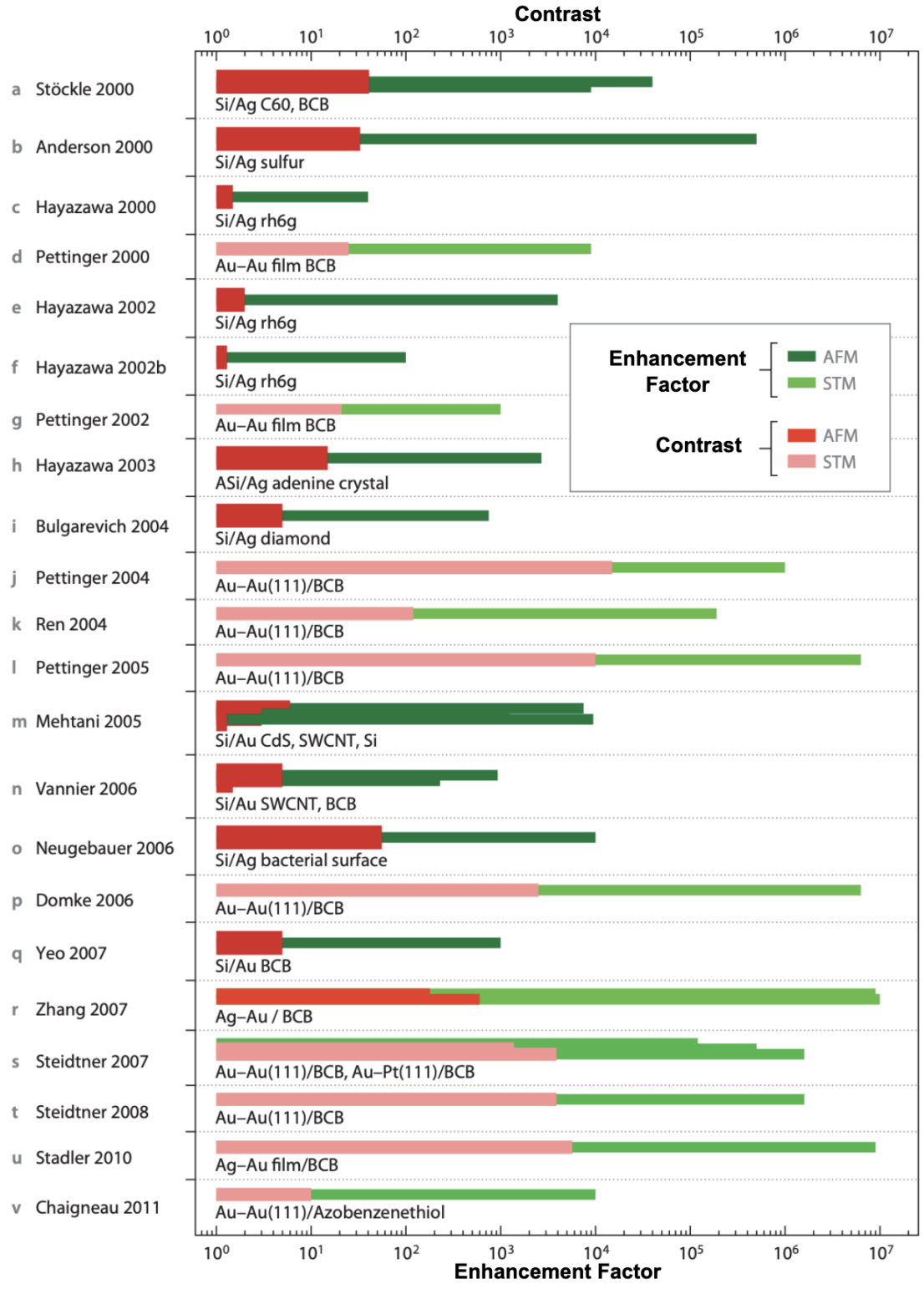

Figure 3 – Several TERS experiment and their calculated Contrast and Enhancement Factor, adapted from Ref. [57]

a Stöckle 2000 [1]; b Anderson 2000 [2]; c Hayazawa 2000 [3]; d Pettinger 2000 [58], e Hayazawa 2002 [59]; f Hayazawa 2002b [60]; g Pettinger 2002 [61]; h Hayazawa 2003 [62]; i Bulgarevich 2004 [52]; j Pettinger 2004 [4]; k Ren 2004 [35]; l Pettinger 2005 [63]; m Mehtani 2005 [55]; n Vannier 2006 [64]; o Neugebauer 2006 [56]; p Domke 2006 [65]; q Yeo 2007 [38]; r Zhang 2010 [66]; s Steidtner 2007 [67]; t Steidtner 2008 [68]; u Stadler 2010 [34]; v Chaigneau 2011 [69].

3 Theoretical Description of a Tip-approach Curve

In this chapter, the function $I_{\Gamma}^{TERS}(\Delta z)$, i.e. the tip-approach curve will be derived, following the steps published in Ref. [29]. The objective is not going through all the numerical calculations, but rather expose the necessary assumptions and physics related phenomena.

3.1 TERS system model

In order to derive $I_{\Gamma}^{TERS}(\Delta z)$, the TERS system is modeled by three components: the nanoantenna, the graphene sample and the detector. The first one is located at $\mathbf{r}' = (0, 0, z)$, the graphene is assumed to occupy all the xy-plane at $(x, y, 0)$ and the detector is located infinitely distant from this plane, as illustrated in Fig.4. This system is illuminated by an incident electric field $\mathbf{E}_0$ of frequency ω. This electric field is assumed to have the component along the nanoantenna's tip E_z greater than the components on the xy-plane, to mimic a radially polarized laser passing through a highly focusing objective lens [23, 26, 27].

The next step is to calculate how these components interact with each other via the scattering process.

3.1.1 Scattered Electric field

The resulting scattered electric field $\mathbf{E}^s$ can be calculated by considering a surface $\mathbb{D}$ (it could also be generalized by a volume) composed by infinitesimally small induced dipoles $\mathbf{p}(\mathbf{r}, \omega)$ and summing all their contribution to the scattered field and a Green's function $\overleftrightarrow{G}$ that represents the evolution of the radiation field from a point $\mathbf{r} \in \mathbb{D}$ to another unspecified point $\mathbf{r}_0 \notin \mathbb{D}$ [70]

$$\mathbf{E}^s(\mathbf{r}_0; \omega) = \frac{\omega_s^2}{\epsilon_0 c^2} \int_{\mathbb{D}} d^2 r \overleftrightarrow{G}(\mathbf{r}_0, \mathbf{r}; \omega) \mathbf{p}(\mathbf{r}; \omega), \tag{3.1}$$

where constants ϵ_0 and c are the vacuum's permittivity and the speed of light, respectively. Each dipole can be written in terms of its polarizability tensor $\overleftrightarrow{\alpha}(\mathbf{r}; \omega)$ and an excitation electric field $\mathbf{E}(\mathbf{r}; \omega)$:

$$\mathbf{p}(\mathbf{r}; \omega) = \overleftrightarrow{\alpha}(\mathbf{r}; \omega) \mathbf{E}(\mathbf{r}; \omega). \tag{3.2}$$

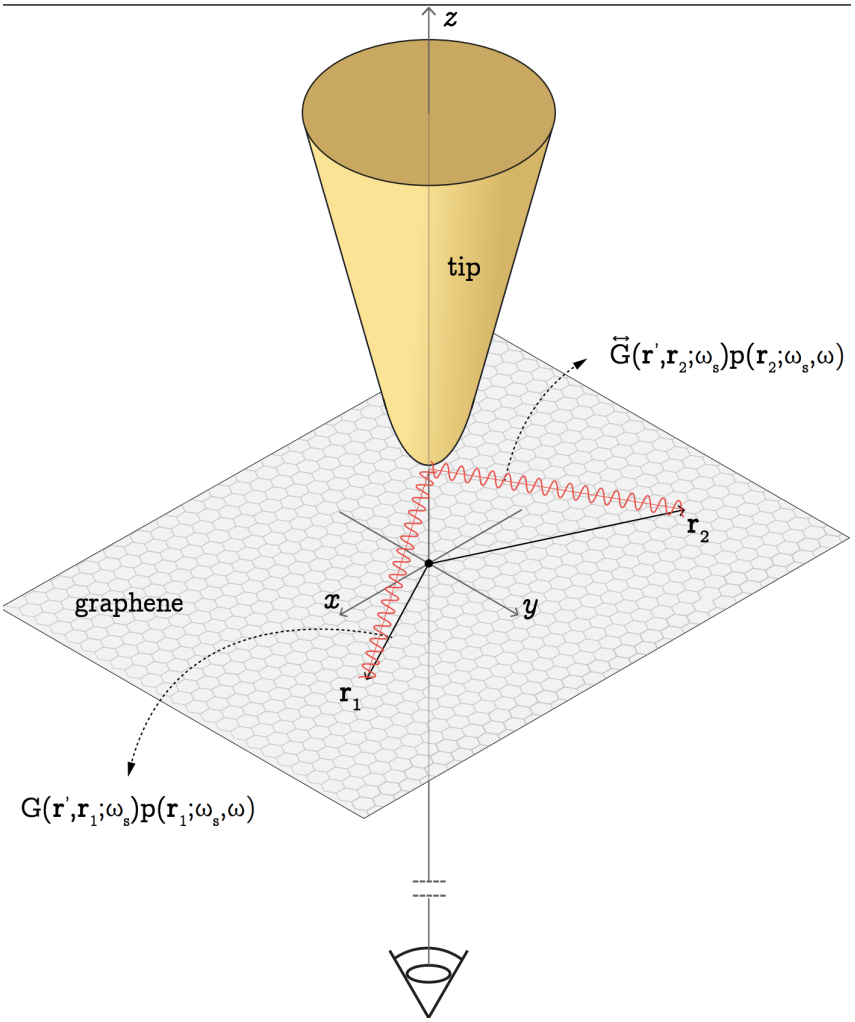

Figure 4 – System of Reference
Spatial distribution of the three components of the TERS model: The nanoantenna's tip is located in $\mathbf{r}' = (0, 0, z)$; the graphene is located in the *xy-plane* $(x, y, 0)$ and the detector located far from the scattering region in $\mathbf{r}_0$. The red waves represent the light scattered from points $\mathbf{r} = \mathbf{r}_1$ and $\mathbf{r} = \mathbf{r}_2$ propagating to the point $\mathbf{r}'$. These two waves might interfere if $|\mathbf{r}_1 - \mathbf{r}_2| \leq L_c$. Adapted from Ref. [29].

3.1.2 Detector

Setting the detector in the position $\mathbf{r}_0$, infinitely distant from the scattered region. The detector is able to measure the intensity associated with the scattered electric field $\mathbf{E}^s(\mathbf{r}_0, \omega_s)$. Only the Raman frequency ω_s is detected. This is the same as assuming that a filter is placed in front of the detector and is able to block all the photons except the ones of frequency ω_s. Then, the detector measures a signal of intensity $I(\mathbf{r}_0, \omega_s)$ which can be calculated as an ensemble average of $\mathbf{E}^s(\mathbf{r}_0, \omega_s)$ in the form

$$I(\mathbf{r}_0, \omega_s) = \langle \mathbf{E}^{s*}(\mathbf{r}_0, \omega_s) \cdot \mathbf{E}^s(\mathbf{r}_0, \omega_s) \rangle \, . \tag{3.3}$$

Inserting Eq.3.1 into 3.3 leads to

$$\begin{aligned} I(\mathbf{r}_0,\omega_s) &= \langle \mathbf{E}^{s*}(\mathbf{r}_0,\omega_s)\cdot\mathbf{E}^{s}(\mathbf{r}_0,\omega_s)\rangle \\ &= \frac{\omega_s^4}{\epsilon_0^2 c^4}\int\limits_{\mathbb{D}} d^2r_1 \int\limits_{\mathbb{D}} d^2r_2 \\ &\left\langle \overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_1;\omega_s)\mathbf{p}(\mathbf{r}_1;\omega_s)\cdot\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_2;\omega_s)\mathbf{p}(\mathbf{r}_2;\omega,\omega_s)\right\rangle. \end{aligned} \tag{3.4}$$

The integrals are evaluated over the sample's domain.

3.1.3 Graphene

Redefining Eq.3.2 in terms of the graphene Raman tensor $\overleftrightarrow{\alpha}^{\gamma}$ describing the symmetry properties of a given mode γ excited by an electric field $\mathbf{E}(\mathbf{r},\omega)$, leads to

$$\mathbf{p}^{\gamma}(\mathbf{r};\omega_s,\omega) = \overleftrightarrow{\alpha}^{\gamma}(\mathbf{r};\omega)\mathbf{E}(\mathbf{r};\omega). \tag{3.5}$$

For the main modes observed in graphene, the $\overleftrightarrow{\alpha}^{\gamma}$ assume the values below (or a linear combination of them [71]) depending on the associated phonon symmetry [72]

$$\overleftrightarrow{\alpha}^{G}(E_{2g1}) = \alpha^{G}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, \qquad \overleftrightarrow{\alpha}^{G}(E_{2g2}) = \alpha^{G}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \qquad \overleftrightarrow{\alpha}^{D,G'}(A_1) = \alpha^{D,G'}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{3.6}$$

We have only considered the in-plane polarizability components because it is well established that graphene does not scatter efficiently for incoming or outgoing light polarized out-of-plane.

The total field $\mathbf{E}$ that excites the graphene results from the summation of incident electric field $\mathbf{E}_0$ and the electric field resulting from the interaction of the incident electric field and the nanoantenna. Before explicitly writing an equation for $\mathbf{E}$ in terms of the nanoantenna scattered field and $\mathbf{E}_0$, another important phenomenon must be considered in the calculation of I: the coherence effects of the graphene phonons, as mentioned in the Sec.1.2. This interference is considered to happen within a Gaussian correlation of length L_c on the form [29]

$$\begin{aligned} I(\mathbf{r}_0,\omega_s) &\propto \langle \alpha^{*\gamma}(\mathbf{r}_1,\omega_s)\alpha^{\gamma}(\mathbf{r}_2,\omega_s)\rangle \\ &= \alpha^{*\gamma}(\mathbf{r}_1,\omega_s)\alpha^{\gamma}(\mathbf{r}_2,\omega_s)\frac{e^{-|\mathbf{r}_1-\mathbf{r}_2|^2/L_c}}{\pi L_c^2}. \end{aligned} \tag{3.7}$$

In the next subsection, the interaction of $\mathbf{E}_0$ with the nanoantenna is addressed.

3.1.4 Nanoantenna

The total electric field $\mathbf{E}$ that excites the graphene can be calculated as

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_0(\mathbf{r},\omega) + \frac{\omega_s^2}{\epsilon_0 c^2}\int_{tip} d^2r' \overleftrightarrow{G}^0(\mathbf{r},\mathbf{r}';\omega_s)\overleftrightarrow{\alpha}_{tip}(\mathbf{r}';\omega)\mathbf{E}_0(\mathbf{r}';\omega). \tag{3.8}$$

where the first term is simply the incident electric field and the second one is the secondary dipole field generated by the nanoantenna excited by $\mathbf{E}_0$, where α_{tip} is the nanoantenna's polarizability tensor. In order to evaluate the integral, two simplifications concerning the nanoantenna's properties can be considered, as in Ref. [23]. The first one comes from the anisotropic polarizability of the nanoantenna

$$\overleftrightarrow{\alpha}(\mathbf{r}') = \begin{bmatrix} \alpha_\perp & 0 & 0 \\ 0 & \alpha_\perp & 0 \\ 0 & 0 & \alpha_\parallel \end{bmatrix}, \tag{3.9}$$

where

$$\alpha_\parallel = 2\pi\epsilon_0 r_{tip}^3 f_e(\omega), \tag{3.10}$$

$$\alpha_\perp = 4\pi\epsilon_0 r_{tip}^3 \frac{\epsilon(\omega)-1}{\epsilon(\omega)+2}. \tag{3.11}$$

Since the component of $\mathbf{E}_0$ along the z direction is the most important one, the contributions from the $\alpha_\perp$ terms are neglected.

The second consideration states that the entire contribution of the nanoantenna (integral over the tip surface) can be approximated as a single dipole located at $\mathbf{r}' = (0,0,z)$, leading to a trivial integration over a single point. Therefore, Eq.3.8 can be rewritten as

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_0(\mathbf{r},\omega) + \frac{\omega_s^2}{\epsilon_0 c^2}\overleftrightarrow{G}^0(\mathbf{r},z;\omega)\overleftrightarrow{\alpha}_{tip}(z;\omega)\mathbf{E}(z;\omega). \tag{3.12}$$

3.1.5 Scattering modes

In the same way that $\mathbf{E}$ takes into account the nanoantenna's effects, the Green's function in Eq.3.4 also does. It can be written in terms of the nanoatenna's components as

$$\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r};\omega_s) = \overleftrightarrow{G}^0(\mathbf{r}_0,\mathbf{r};\omega_s) + \frac{\omega_s^2}{\epsilon_0 c^2}\overleftrightarrow{G}^0(\mathbf{r}_0,\mathbf{r};\omega_s)\overleftrightarrow{\alpha}_{tip}(z;\omega_s)\overleftrightarrow{G}^0(z,\mathbf{r};\omega_s). \tag{3.13}$$

The first term in the right-hand side accounts for the field that propagates directly to the detector. The second accounts for the scattered modes generated by multiple scattering events between the graphene and the nanoantennas's tip.

By grouping all elements introduced up to now, Eq.3.4 can finally be rewritten in a matrix representation form as:

$$\begin{aligned} I(\mathbf{r}_0,\omega_s) =& \frac{\omega_s^4}{\epsilon_0^2 c^4}\sum_{l,m,n}\sum_{i,j}\iint_{-\infty}^{\infty} dx_2 dy_2 G_{ln}(-z_\infty;x_2,y_2;\omega_s)\alpha_{nj}^{\gamma}E_j(x_2,y_2,\omega) \\ &\times \iint_{-\infty}^{\infty} dx_1 dy_1 \frac{e^{[(x_1-x_2)^2+(y_1-y_2)^2]/Lc^2}}{\pi L c^2} G_{lm}^{*}(-z_\infty;x_2,y_2;\omega_s)\alpha^{\gamma *}E_i^{*}(x_1,y_1,\omega). \end{aligned} \tag{3.14}$$

The introduction of the terms derived in 3.1.3 and 3.1.4 gives

$$\begin{aligned} G_{ln}(-z_\infty;x_2,y_2;\omega_s)\alpha_{nj}^{\gamma}E_j(x_2,y_2,\omega) =& \; G_{ln}^{0}(-z_\infty;x,y;\omega_s)\alpha_{nj}^{\gamma}(x,y,\omega)E_{0j}(x,y;\omega) \\ &+ \frac{\omega_s^2}{\epsilon_0 c^2}G_{lz}^{0}(-z_\infty;z';\omega_s)\alpha_{\parallel}(\omega_s)G_{zn}^{0}(z';x,y;\omega_s)\alpha_{nj}^{\gamma}(x,y;\omega)E_{0j}(x,y;\omega) \\ &+ \frac{\omega_s^2}{\epsilon_0 c^2}G_{ln}^{0}(-z_\infty;x,y;\omega_s)\alpha_{nj}^{\gamma}(x,y;\omega)G_{jz}^{0}(x,y;z';\omega)\alpha_{\parallel}(\omega)E_{0j}(x,y,\omega) \\ &+ \frac{\omega^2}{\epsilon_0 c^2}G_{ln}^{0}(-z_\infty;z';\omega_s)\alpha_{\parallel}(\omega_s)G_{zn}(z';x,y;\omega_s)\alpha_{nj}^{\gamma}(x,y;\omega) \\ &\times G_{jz}^{0}(x,y;z';\omega)\alpha_{\parallel}E_{0z}(z';\omega) \end{aligned} \tag{3.15}$$

The first term corresponds to the S mode, which is related to the conventional Raman far-field scattering; it has no relation with any nanoantenna's component whatsoever. The second term accounts for the photons scattered by the nanoantenna and subsequently scattered by the graphene, namely ST. The third one inverts this order, TS: first the graphene scattering and then the nanoantenna. Finally, the last term is the light scattered from the tip to the graphene, and then back the nanoantenna, TST. The scattering modes are illustrated in Fig.5

The S mode and the TS mode are assumed here to be orders of magnitude weaker than the TST and ST modes, and they will be neglected in the present work, following the procedure of Ref. [29].

3.1.6 The Simplified Tip-approach equation

After taking care of all the assumptions and considerations explained in the last subsections, the integral is tricky and requires taking Fourier transformation and approximations. The calculation can be implemented in a *Mathematica*® environment, but for a better understanding of how the experimental parameters play a role, the result can be evidenced in a simplified equation [48]

$$I_{\Gamma}^{TERS}(\Delta z) = C_{\Gamma} f_e^2 r_{tip}^6 \left(f_e^2 r_{tip}^6 g_{\Gamma}^{TST}(L_c,z) + g_{\Gamma}^{ST}(L_c,z) \right). \tag{3.16}$$

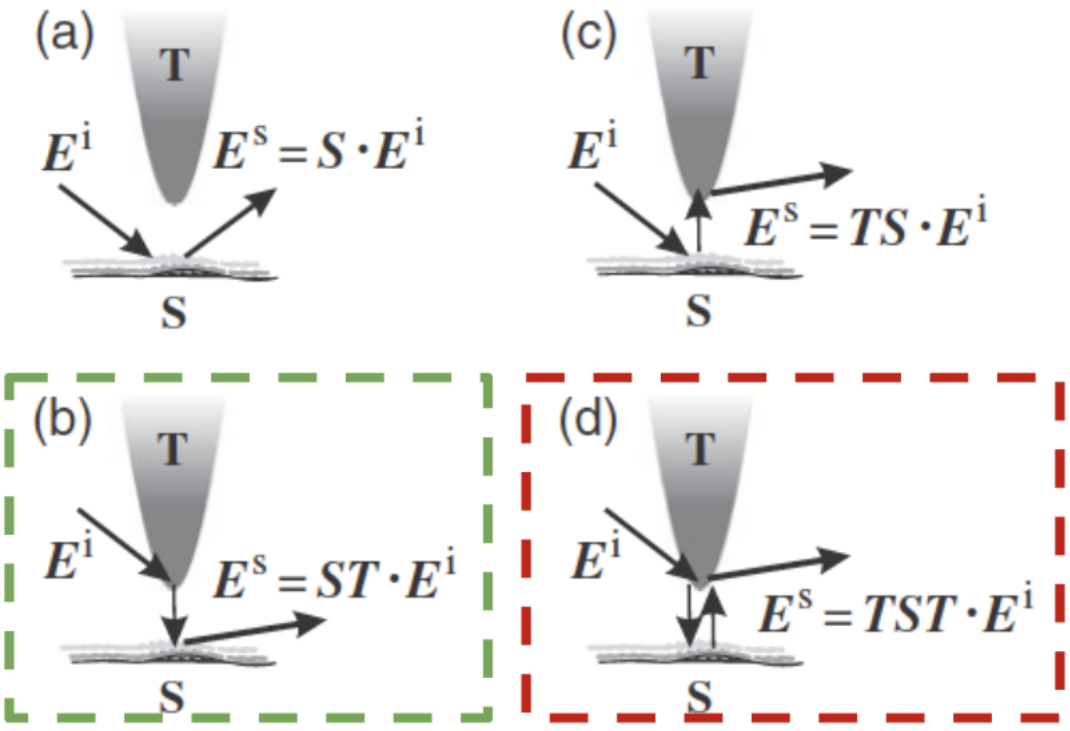

Figure 5 – Scattering Modes

Adapted from [23], the illustrations shows the scattering modes: a) The S mode - The nanoantenna plays no role in the scattering; b) The TS mode - light scatters form the sample to the nanoantenna's tip; c) The ST mode - light scatters from the nanoantenna's tip to the sample) The TST mode - light scatters from the nanoantenna's tip then to he sample and again to the nanoantenna's tip. The highlights in the ST and TST modes emphasizes they are order of magnitude greater than the other two.

Here we just want to stretch out that I_Γ^{TERS} has two contributions: the functions g^{TST} and g^{ST} related to the scattering modes TST and ST, respectively. C_Γ accounts for the Raman cross section of the Γ symmetry mode and several other factors not explicitly considered in this formulation, such as detector's sensitivity, laser intensity and system alignment.

Equation 3.16 already brings the important ingredients for describing the relation between the tip-approach curve and the experimental parameters of interest, including f_e. In order to get a better understanding of Eq.3.16 and how to use it in a protocol to determine f_e, it is important to understand how Γ, L_c, r_{tip} and f_e influence the tip-approach curve. The next section will be dedicated to some results.

3.2 Study of the parameters

For studying the Γ and L_c influence on $I_\Gamma^{TERS}(\Delta z)$, consider the fixed parameters $f_e = 5$, $r_{tip} = 15nm$ and $z_0 = 3nm$ (see Table 2). Figures 6a,b show the graphs for different

symmetry modes mixing the A_{1g} and E_{2g} symmetries for a fixed value of $L_c = 30nm$. These symmetries were chosen to simulate the results from graphene, where the Raman peaks G and G' bands are related to the E_{2g} and A_{1g} symmetry modes, respectively [71]. Figure 6a shows the absolute $I_\Gamma^{TERS}(\Delta z)$ in a log-log plot for $C_\Gamma = 1$. In Fig. 6b the same result is displayed but normalized at $\Delta z = 0$.

In the normalized picture, the faster the decay of the intensity as Δz increases, the larger the effective enhancement. The A_{1g} mode exhibits the larger enhancement, in agreement with [28, 29, 71]. Now, fixing the symmetry mode and varying the L_c, using the same tip parameters as in Fig. 6a,b, Figs.6c and 6d show the tip-approach curves for the A_{1g} and E_{2g} symmetry modes, respectively (parameters summarized in Table 2). For the A_{1g} mode (G band), Fig. 6c, from $L_c \to 0$ to $L_c = 80nm$, indicated by the dark-blue line and yellow lines, respectively, the dependence with Δz are smother, showing that the ST scattering mode plays a more important role in these scattering events. By increasing the L_c value further, represented by the orange and red lines, the curves changes their behavior with Δz, decaying faster for smaller values of Δz , showing that the TST mode is more relevant after L_c values between 30 and 80nm, tending to the red curve for $L_c \to \infty$.

Figure 6d shows the same plot for the E_{2g} mode. Again, the slope of the curves indicates that the ST scattering gets more relevant for larger values of L_c. For $L_c \to \infty$, the curve approaches to a constant value, indicating no tip-induced enhancement for the fully coherent E_{2g} scattering due to destructive interference, in agreement with [28, 29, 71].

The other two parameters explored were the r_{tip} and f_e. Figure 6e shows that, for Δz greater than 5nm the smaller the r_{tip}, greater is the enhancement, in accordance with Ref. [23]. Although, for a $\delta z \to 0$ the model shows that there is an optimal r_{tip} around 5nm, as the dark-green light is greater than the yellow and dark-blue lines (r_{tip}equals 10 and 2nm, respectively). In terms of effective of the enhancement, figure 6f shows no clear distinction among r_{tip} greater than 10 nm, yet, for r_{tip} equals 2 and 5 nm, the models predicts a slower decay. Currently, it is not known by the authors is there is experimental data to validate the model for Δz or r_{tip} below 5 nm, and could be the theme for future work. On the other hand, the model shows the expected behavior when increasing f_e results in more absolute enhancement and larger the effective enhancement (as in Fig. 6g and h, respectively).

The theory described in this section can now be used in a protocol to estimate f_e. It is crucial to have the z_0 and r_{tip} values well established and the sample properties L_c and Γ well defined. For this reason, it is important to have a reference material. To avoid the instrumental aspects hidden in C_Γ, it is also important to work with the normalized data. After a normalized tip-approach curve data is extracted from such a reference material, the protocol includes adjusting the data by choosing the value of f_e that better describes the normalized $I_\Gamma^{TERS}(\Delta z)$ curve. A graphene device is proposed here for the reference

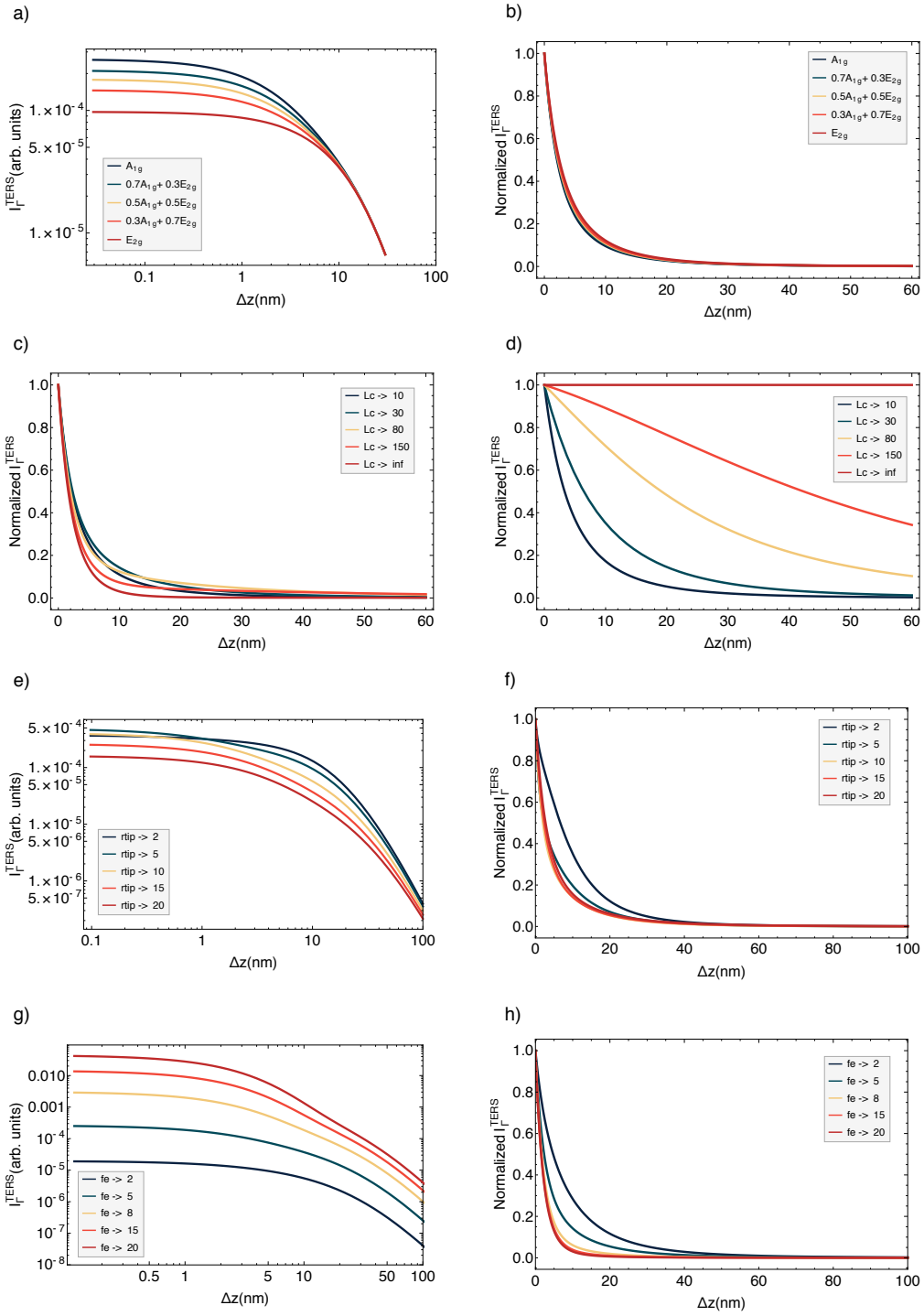

Figure 6 – All graphics represent the tip-approach curves - $I_\Gamma^{TERS}(\Delta z)$ as a function of tip-displacement Δz. In each graph, one parameter is variable and all other parameters fixed, as summarized in Table 2. (a, e, g) are log-log plots, while the others exhibit a linear scale and are normalized. The normalization process consists in dividing each curve by its value at $\Delta z = 0$. Results first published in Ref. [48]

Table 2 – Parameters used to plot the graphics in Fig.6

Panel	Scale	Fixed parameters	emphasized parameter
a)	Log-log	$f_e = 5$ $r_{tip} = 15$ nm $z_0 = 3$ nm $L_c = 30$ nm	phonon symmetry Γ
b)	Linear	$f_e = 5$ $r_{tip} = 15$ nm $z_0 = 3$ nm $L_c = 30$ nm	phonon symmetry Γ
c)	Linear	$f_e = 5$ $r_{tip} = 15$ nm $z_0 = 3$ nm $\Gamma = A_{1g}$	phonon coherence length L_c
d)	Linear	$f_e = 5$ $r_{tip} = 15$ nm $z_0 = 3$ nm $\Gamma = E_{2g}$	phonon coherence length L_c
e)	Log-log	$f_e = 5$ $z_0 = 3$ nm $L_c = 30$ nm $\Gamma = A_{1g}$	tip apex radius r_{tip}
f)	Linear	$f_e = 5$ $z_0 = 3$ nm $L_c = 30$ nm $\Gamma = A_{1g}$	tip apex radius r_{tip}
g)	Log-log	$r_{tip} = 15$ nm $z_0 = 3$ nm $L_c = 30$ nm $\Gamma = A_{1g}$	enhancement factor f_e
h)	Linear	$r_{tip} = 15$ nm $z_0 = 3$ nm $L_c = 30$ nm $\Gamma = A_{1g}$	enhancement factor f_e

material because graphene is a widely studied 2D material [73] with a well-established L_c [28].

4 The Calibration Device and Measurement Protocol

The proposed protocol to measure the intrinsic enhancement factor f_e is described in this chapter. It is based on the fitting of the tip-approach curve $I_\Gamma^{TERS}(\Delta z)$ which depends on the tip-sample distance Δz. Figure 7 defines the relevant parameters. Considering the dipole located at the center of the sphere representing the tip apex, the total dipole-sample distance is given by $z = r_{tip} + z_0 + \Delta z$, where r_{tip} is the tip-apex sphere radius, and z_0 is the minimum distance achieved by the tip from the sample, as defined by the SPM setpoint. Γ and L_c are the phonon symmetry and coherence length, respectively.

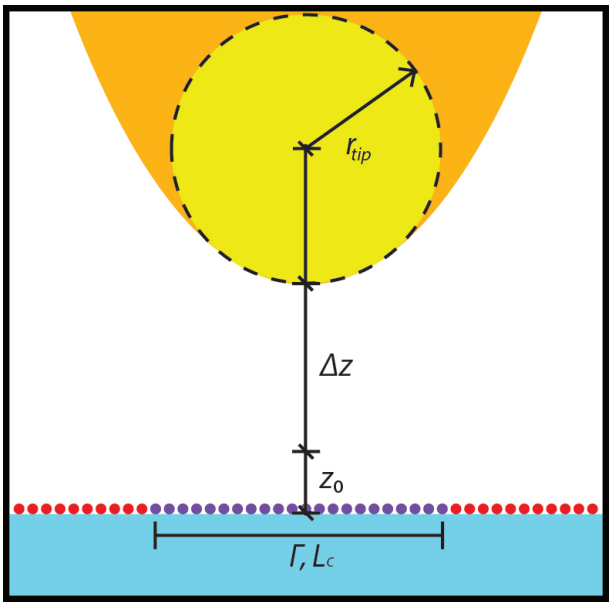

Figure 7 – Tip-approach parameters diagram
Illustration of the relevant parameters for the definition of $I_\Gamma^{TERS}(\Delta z)$. The dipole excitation is located at the center of a sphere (yellow) with radius r_{tip}, at the apex of the nanoantenna (orange). The total dipole-sample distance is $z = z_0 + \Delta z + r_{tip}$, where Δz is the displacement from the initial setpoint z_0 in a tip-sample approach curve. The sample is represented by the small dots at the substrate on surface. Given a phonon with symmetry Γ, the light field scattered by the region smaller then the phonon coherence length L_c (purple dots) exhibits interference. Outside this region (red dots), the vibrations in the material system are uncorrelated. Image first published in Ref. [48]

The experimental data is acquired by measuring the Raman intensity S scattered by a graphene in a TERS system, for different tip-sample positions. Due to the difficulty and lack of standardized procedure to do this experiment, a calibration device is proposed to increase the reproducibility of the f_e determination.

In the following sections, the calibration device and referential material will be discussed as a standard material to measure f_e. Finally, the protocol itself will be described.

4.1 Fitting the Tip-approach Curves

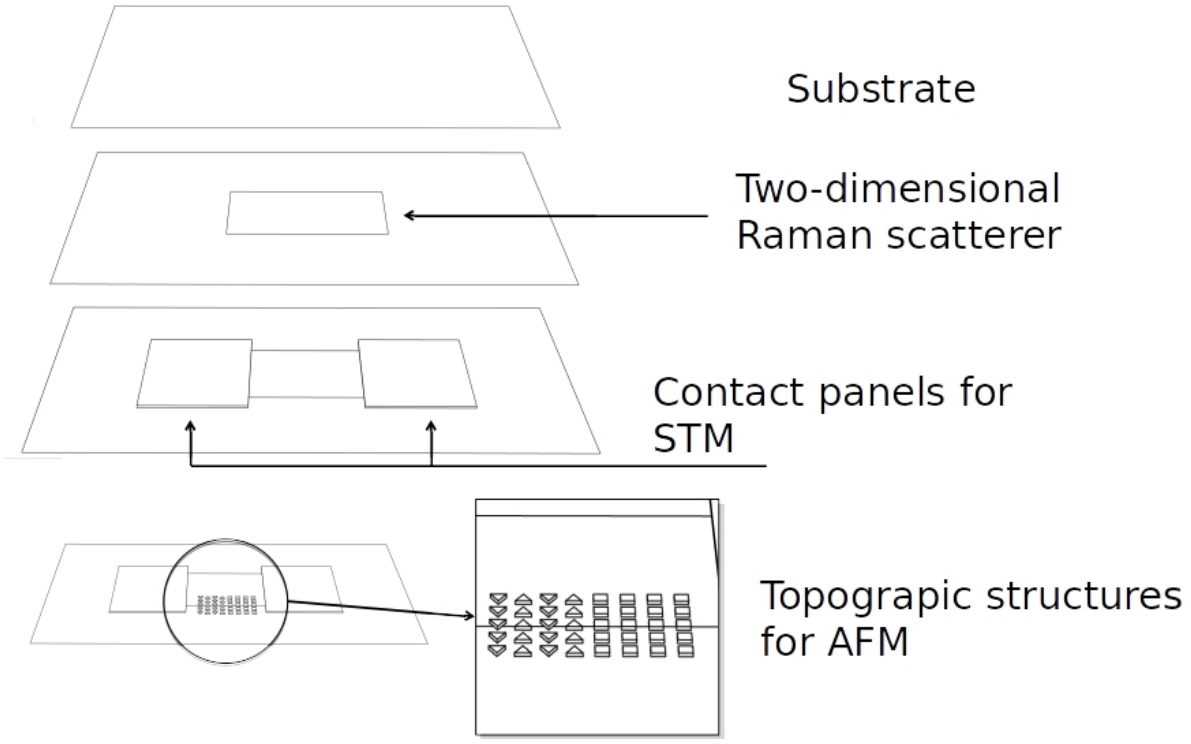

Figure 8 – Schematics illustrates the TERS calibration device
Adapted from [48] and [74]. From top to bottom: 1) Transparent substrate; 2) Pristine monolayer graphene; 3) Contact panels made with conducting material for STM; 4) Topographic structures with different geometries for AFM calibration with well-defined edges.

The theory described in chapter 3 can now be used in a protocol to estimate f_e, as evidenced by the results displayed in Fig. 6. It is crucial to have the z_0 and r_{tip} values well established and the sample properties L_c and Γ well defined. For this reason, it is important to have a reference material. To avoid the instrumental aspects hidden in C_Γ, it is also important to work with the normalized data. After a normalized tip-approach curve data is extracted from such a reference material, the protocol includes adjusting the data by choosing the value of f_e that better describes the normalized $I_\Gamma^{TERS}(\Delta z)$ curve. A graphene-based callibration device is proposed here, because graphene is a widely studied 2D material [73] with a well-defined L_c [28].

4.2 Calibration Device

A few parameters have to be calibrated in a TERS system in order to be able to obtain the best performance of a nanoantenna. These parameters influence the accuracy of the Raman spectral frequencies, spectral imaging and scanning probe imaging. The proposed calibration device intendsto be a one-stop source to align most moving parts of the system (see Fig. 8). The same platform can then be used to obtain f_e and, subsequently, to perform repeatable or traceable TERS experiments *in situ*. The components of the calibration device are described in Fig. 8, which comprises:

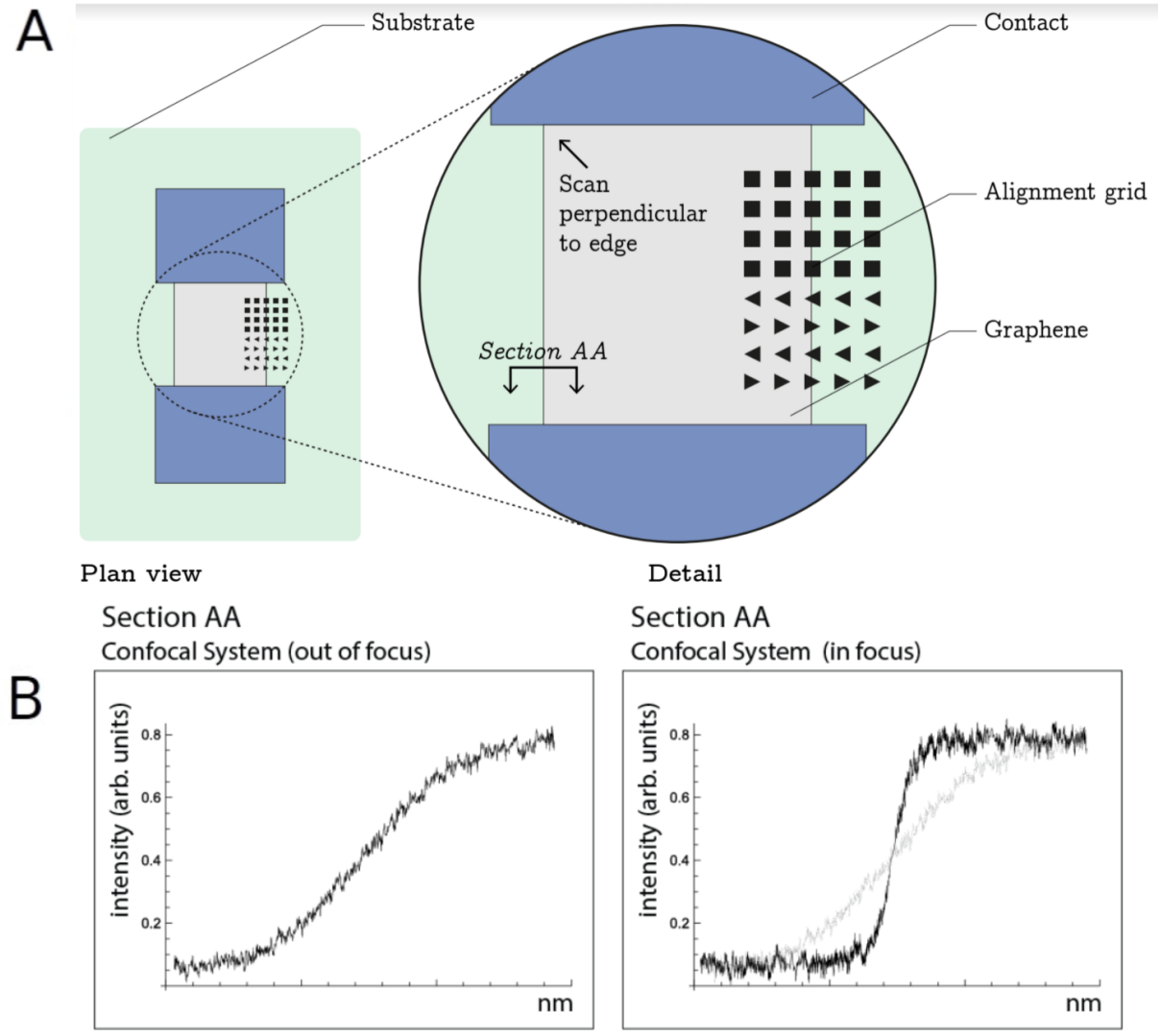

Figure 9 – Schematics of the calibration device
Adapted from [48] and [74] The detailed view of A illustrates how the lateral scan must be performed to maximize the SNOM lateral resolution. B illustrates a misaligned system before calibration (left) and the expected signal after alignment in comparison with the first (right).

- **Substrate**: The substrate has to be a material with no optical response in the same range of a graphene Raman scatters. Glass, fused quartz or other transparent media

are ideal for sample illumination from the bottom.

- **Two-dimensional Raman scatterer**: graphene is the reference material utilized for confocal and TERS calibration.
- **Contact panels for STM**: in the case SPM is utilized in the STM mode, for current drain.
- **Topographic arrangement for AFM**: periodic topographic structure with well-defined dimensions for calibration of the SPM system in the AFM mode.

4.3 Experimental Protocol

4.3.1 Alignment of Scanning Confocal Optical Microscope

The scanning confocal optical microscope (SCOM) is the first system to be aligned and, in order to do that, it is ideal to have a sample that provides a strong Raman signal, with well-defined topological edges. This alignment is performed by moving the laser beam in and out of this reference material to maximize the optical lateral (X/Y) resolution (Fig.9).

The proposed platform provides a single layer graphene for the alignment of the SCOM system. Graphene is considered a reference material for this step, due to its strong Raman signal and because it is a truly bidimensional (one atom thick) structure. By scanning in and out of the graphene sheet, the user is able to maximize the lateral resolution. This can be directly observed through the slope of the signal generated when the laser moves in and out of the graphene sheet. The steeper the slope, the better the optical lateral resolution of the system (Fig. 9B). The expected resolution for a good, well-aligned optical system is below half the wavelength of the excitation laser being used.

4.3.2 Configuration of Scanning Probe Optical Microscope

With the SCOM aligned, the second step is to configure the scanning probe optical microscopy (SPOM) system. The requirement in this step is to align the tip apex with the laser focal point. The proposed platform provides a topographic alignment grid designed not only to facilitate this task (see Fig.10), but also to allow a quick and effective in-situ inspection of the tip apex radius.

To provide meaningful information for tip alignment, the grid has both square and triangular shapes. By using the triangular shapes as reference for alignment, it is possible to have relative tip-focus position information along both X and Y directions while scanning only in one direction (Fig. 10). By scanning over the topographic grid and comparing the generated confocal and topographical images, the user can see how

misaligned the tip is with respect to the focal point, and to adjust the tip position in real time. The geometrical nature of the topographic grid facilitates the alignment procedure, by providing the user an easy way to calculate the tip-to-focus offset direction and distance.

In addition, this procedure can be used to evaluate the r_{tip}. This parameter is crucial to minimize the error in estimating the f_e by fitting the tip-approach curve.

4.3.3 Tip-approach Procedure

After the alignment of both SCOM and SPOM using this protocol, and having the r_{tip} characterized by the tip lateral resolution, the next step is finally the determination of f_e for a given phonon mode Γ.

The tip-approach curve consists in measuring the Raman Intensity $I_{\Gamma}^{TERS}(\Delta z)$ for different Δz values in the same XY position of the reference standard material (on top of graphene). The experimentally obtained $I_{\Gamma}^{TERS}(\Delta z)$ data is then fitted with the model described in chapter 3, using $Mathematica^{®}$ (see APPENDIX B).

The absolute I_{Γ}^{TERS} values depend on the defined C_{Γ}, which includes instrumental aspects, such as distance to the detector. To eliminate this factor from our analysis, the signal should be normalized. All points must be divided by the Raman intensity obtained at $\Delta z = 0$. The f_e value is the one that better adjusts the tip-approach curves.

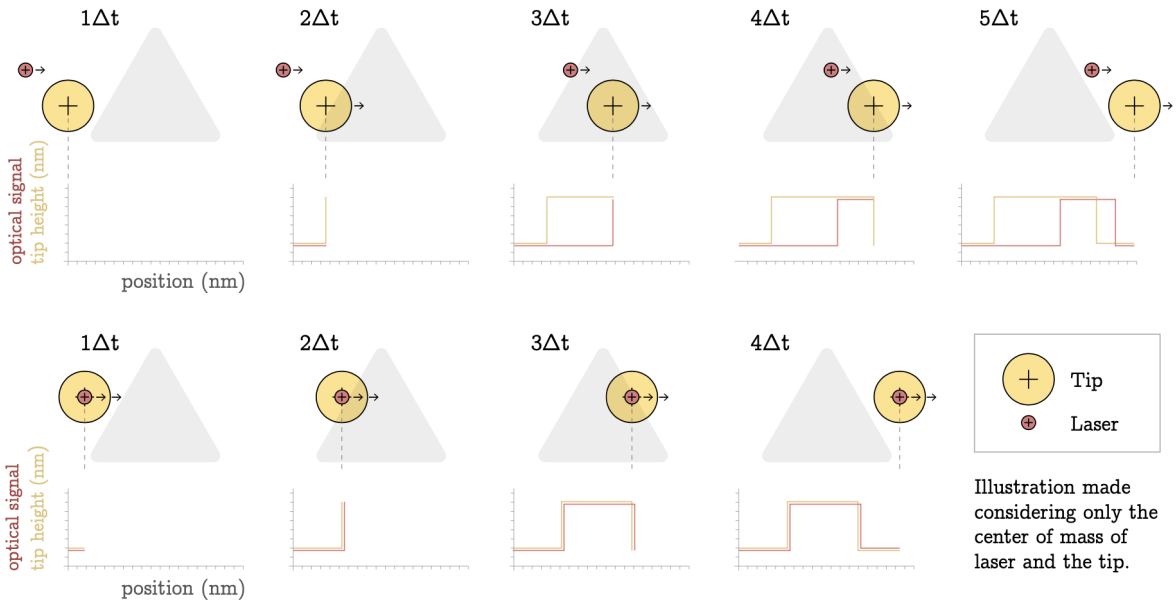

Figure 10 – Scanning of a topographic structure

Adapted from [48] and [74] This figure illustrates the scanning of topographic structures (gray triangle), considering (top) the situation when the tip SPM (yellow circle) is misaligned with the laser spot (red circle), and (bottom) the situation when the tip and optical signal are aligned. The expected measured line profiles are below the figures. For the AFM tip, it is a measure of the tip height as a function of the lateral position. For the optics, it represents the measured signal intensity as function of the lateral position. By measuring both optical signal and tip-height as a function of the lateral movement, it is possible to infer whether the tip is aligned with the optical signal in the XY plane.

5 Conclusion

As described in the Nanoantenna Fabrication subsection, there are several methods of producing nanoantennas. Currently, the definition of enhancement factor EF varies between different research groups, making it hard to access whether a tip is better than another. The protocol and calibration device described here intend to solve this problem. It is based on previous work that considers the nanoantennas intrinsics enhancement factor f_e, see Chap.3, and it can be inferred by adjusting a tip-approach curve, as detailed in Chap.4.

The parameters influencing the tip-approach curve shape were discussed in Sec.3.2 and summarized in Fig 6. Both sample properties, Γ and L_c, and tip properties r_{tip} and f_e, influence the behavior of the tip-approach curve. In order to measure the f_e, the parameters Γ, L_c and r_{tip}, must be measured. Therefore, the calibration device described in Sec.4.2 is a graphene-based device with well-defined Γ and L_c, to check r_{tip} and check the whole alignment of the system, by following the experimental protocol also described in Sec.4.2.

The methodology presented in this dissertation has already been used by TERS community. The first one is the *Vasconcelos et al.* article (Ref. [42]). There the authors show a new method of fabricating nanopyramid-shaped tip with tunable LSPR conditions. In order to access the f_e of these tips, this protocol was followed resulting in a $f_e \approx 9$. This value is easily comparable with the result from Ref. [28] that found $f_e \approx 4$ for a nanoantenna made using another fabrication process.

The second contribution was to *Alencar et al.* (Ref. [75]). The main goal of the article was to infer the L_c of GaS using similar procedure. In order to do so, the TS mode was included in the tip-approach curve (not part of this dissertation) and used to adjust experimental data obtained in a TERS experiment. It resulted in a measured $f_e \approx 3.5$ and the consistent $L_c \approx 64$.

With this dissertation, we hope to help minimizing the arbitrariness of the definition of f_e for a TERS nanoantenna, and to provide an useful guide to measure it, providing a value to compare nanoantennas by means of their enhancements, and also enabling deriving other properties of the material, such as L_c.

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Appendix

APPENDIX A – Radialy Pollarized Light and a High Numerical Apperture Lens

A.0.1 Theory and Calculations

A.0.2 Experimental Implementation

APPENDIX B – Tip-approach curve fitting in *Mathematica*®

In this Appendix it is shown the *Mathematica*® implementation of the tip-approach curve and the fit process as part of the measurement protocol proposed in Chapter 4.

B.0.1 Numerical Solutions to Tip-approach Curve Intensities

The Eq.3.14 can be evaluated for G and G' modes and their associated phonon symmetries A_1, E_{2g1} and E_{2g2}, as well as for the scattering modes TST and ST, by solving the integral (38), (39), (51) and (52) of Ref. [29]. The *Mathematica*® implementation of these four equation is written below:

```
a = .74;
b = 4;
c = .08;
d = 1.5;
ap = 0.78;
bp = 2.4;
cp = .18;
dp = .56;

hxx[z_]:=
(a/b^3 (2b - kx^2 z^2) Exp[-(kx^2 + ky^2) z^2 * (4b)^-1] + c/d^3 (2d - kx^2 z^2) Exp[-(kx^2 + ky^2) z^2 * (4d)^-1])
hxy[z_]:= (a/b^3 kx ky z^2 Exp[-(kx^2 + ky^2) z^2 * (4b)^-1] + c/d^3 kx ky z^2 Exp[-(kx^2 + ky^2) z^2 * (4d)^-1])

hyy[z_]:=
(a/b^3 (2b - ky^2 z^2) Exp[-(kx^2 + ky^2) z^2 * (4b)^-1] + c/d^3 (2d - ky^2 z^2) Exp[-(kx^2 + ky^2) z^2 * (4d)^-1])

hx[z_]:=kx (ap/bp^2 Exp[-(kx^2 + ky^2) z^2/(4bp)] + cp/dp^2 Exp[-(kx^2 + ky^2) z^2/(4dp)])

hy[z_]:=ky (ap/bp^2 Exp[-(kx^2 + ky^2) z^2/(4bp)] + cp/dp^2 Exp[-(kx^2 + ky^2) z^2/(4dp)])

fxxxx = Integrate[hxx[z] * hxx[z] * Exp[-(kx^2 + ky^2) Lc^2/4], {kx, -Infinity, Infinity},
{ky, -Infinity, Infinity}];

fxxyy = Integrate[hxx[z] * hyy[z] * Exp[-(kx^2 + ky^2) Lc^2/4], {kx, -Infinity, Infinity},
```

```
{ky, -Infinity, Infinity}];

fxyxy = Integrate [hxy[z] * hxy[z] * Exp [- (kx^2 + ky^2) Lc^2 / 4], {kx, -Infinity, Infinity},
{ky, -Infinity, Infinity}];

lxx = Integrate [hx[z] * hx[z] * Exp [- (kx^2) Lc^2 / 4], {kx, -Infinity, Infinity}]
fxxxx = "0.165318"/("1."Lc^2+"0.916667"(rtip+z+z0)^2) + "0.860168"Lc^4/(2Lc^2+(rtip+z+z0)^2)^3+
"0.107521"Lc^2(rtip+z+z0)^6/("2."Lc^2+(rtip+z+z0)^2)^5 - "0.0268803"(rtip+z+z0)^8/("2."Lc^2+(rtip+z+z0)^2)^5+
"0.0635455"/("1."Lc^2+"1.33333"(rtip+z+z0)^2)+
(rtip + z + z0)^4
("0.000728675"/
(Sqrt["0.0795775"Lc^2 + "0.072946"(rtip + z + z0)^2]
("0.25"Lc^2 + "0.229167"(rtip + z + z0)^2)^(5/2)) +
"0.000746911"/
(Sqrt["0.0795775"Lc^2 + "0.106103"(rtip + z + z0)^2]
("0.25"Lc^2 + "0.333333"(rtip + z + z0)^2)^(5/2)) + "0.322563"Lc^4/(2Lc^2+(rtip+z+z0)^2)^5 -
"0.430084"Lc^2/(2Lc^2+(rtip+z+z0)^2)^4 + "0.188162"/(2Lc^2+(rtip+z+z0)^2)^3) +
(rtip + z + z0)^2
(-("0.00534362"/
(Sqrt["0.0795775"Lc^2 + "0.072946"(rtip + z + z0)^2]
("0.25"Lc^2 + "0.229167"(rtip + z + z0)^2)^(3/2))) -
"0.00298764"/
(Sqrt["0.0795775"Lc^2 + "0.106103"(rtip + z + z0)^2]
("0.25"Lc^2 + "0.333333"(rtip + z + z0)^2)^(3/2)) - "0.430084"Lc^4/(2Lc^2+(rtip+z+z0)^2)^4+
"0.645126"Lc^2/(2Lc^2+(rtip+z+z0)^2)^3);

fxyxy =
2(rtip + z + z0)^4
("0.0000525005"/("0.25"Lc^2+"0.125"(rtip+z+z0)^2)^3 + "0.000215257"/("0.25"Lc^2+"0.229167"(rtip+z+z0)^2)^3+
"0.000220644"/("0.25"Lc^2+"0.333333"(rtip+z+z0)^2)^3);

fxxyy = "0.00112373"
(("299.346"Lc^4 + "565.106"Lc^2(rtip + z + z0)^2+
"240.94"(rtip + z + z0)^4)/
```

$$\Big(\text{“1.”}\mathrm{Lc}^6 + \text{“2.75”}\mathrm{Lc}^4(\mathrm{rtip}+z+\mathrm{z0})^2 + \text{“2.34722”}\mathrm{Lc}^2(\mathrm{rtip}+z+\mathrm{z0})^4 + \text{“0.611111”}(\mathrm{rtip}+z+\mathrm{z0})^6\Big) + (\mathrm{rtip}+z+\mathrm{z0})^2 \left(-\left(\text{“1.68696”}/\left(\sqrt{\text{“0.0795775”}\mathrm{Lc}^2 + \text{“0.0397887”}(\mathrm{rtip}+z+\mathrm{z0})^2}\left(\text{“0.25”}\mathrm{Lc}^2 + \text{“0.125”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^{3/2}\right)\right) - \text{“4.75525”}/\left(\sqrt{\text{“0.0795775”}\mathrm{Lc}^2 + \text{“0.072946”}(\mathrm{rtip}+z+\mathrm{z0})^2}\left(\text{“0.25”}\mathrm{Lc}^2 + \text{“0.229167”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^{3/2}\right) - \text{“2.65868”}/\left(\sqrt{\text{“0.0795775”}\mathrm{Lc}^2 + \text{“0.106103”}(\mathrm{rtip}+z+\mathrm{z0})^2}\left(\text{“0.25”}\mathrm{Lc}^2 + \text{“0.333333”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^{3/2}\right)\right) + (\mathrm{rtip}+z+\mathrm{z0})^4 \left(\frac{\text{“5.98013”}}{\left(\text{“1.”}\mathrm{Lc}^2+\text{“0.5”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^3} + \frac{\text{“24.5191”}}{\left(\text{“1.”}\mathrm{Lc}^2+\text{“0.916667”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^3} + \frac{\text{“25.1327”}}{\left(\text{“1.”}\mathrm{Lc}^2+\text{“1.33333”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^3}\right);$$

$$\mathrm{lxx} = \frac{\text{“0.460876”}}{\left(\text{“1.”}\mathrm{Lc}^2+\text{“0.833333”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^2} + \frac{\text{“3.90696”}}{\left(\text{“1.”}\mathrm{Lc}^2+\text{“2.20238”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^2} + \frac{\text{“8.28005”}}{\left(\text{“1.”}\mathrm{Lc}^2+\text{“3.57143”}(\mathrm{rtip}+z+\mathrm{z0})^2\right)^2};$$

$$\mathrm{E2TST} = \mathrm{rtip}^6\mathrm{fenh}^2\frac{9*\mathrm{rtip}^6*\mathrm{fenh}^2}{32*(z+\mathrm{rtip}+\mathrm{z0})^8}\frac{1}{2}(\mathrm{fxxxx} - \mathrm{fxxyy} + 2\mathrm{fxyxy});$$

$$\mathrm{A1TST} = \mathrm{rtip}^6\mathrm{fenh}^2\frac{9*\mathrm{rtip}^6*\mathrm{fenh}^2}{32*(z+\mathrm{rtip}+\mathrm{z0})^8}(\mathrm{fxxxx} + \mathrm{fxxyy});$$

$$\mathrm{E2ST} = \mathrm{rtip}^6\mathrm{fenh}^2 0.5\mathrm{lxx};$$

$$\mathrm{A1ST} = \mathrm{rtip}^6\mathrm{fenh}^2 0.5\mathrm{lxx};$$

B.0.2 TERS measurement Fit

In order to illustrate the fitting process, the data reported in published in [28] was fitted using the formulas derived in the previous subsection. It consists in Tip-approach curve for the graphene G'-band. The objective was to infer the f_e of the tip given the known parameters: $z_0 = 5nm$, $L_c = 33$ and $r_{tip} = 20$.

$$\mathrm{dadosPRLraw} = \mathrm{Import}[\text{“gp\_center.txt”}, \text{“Table”}];$$

```
zdist = dadosPRLraw[[All, 1]] − dadosPRLraw[[1, 1]];
Gplist = dadosPRLraw[[All, 2]]/dadosPRLraw[[−1, 2]];
Gptable = Table[{(zdist[[i]]), Gplist[[i]]}, {i, 1, Length[zdist]}];


Model = ((A1TST + A1ST)/.{z0 → 5, Lc → 33, rtip → 20})/((A1TST + A1ST)/.{z0 → 5, Lc → 33, rtip -
GPRLfitgust = FindFit[Gptable, Model, {{fenh, 4}}, z]

{fenh → "4.06411"}

Show[ListPlot[Gptable, PlotRange → All],
Plot[Evaluate[Model/.GPRLfitgust], {z, 0, 60}, PlotStyle → Directive[Thick, Red], PlotRange → All]]
```

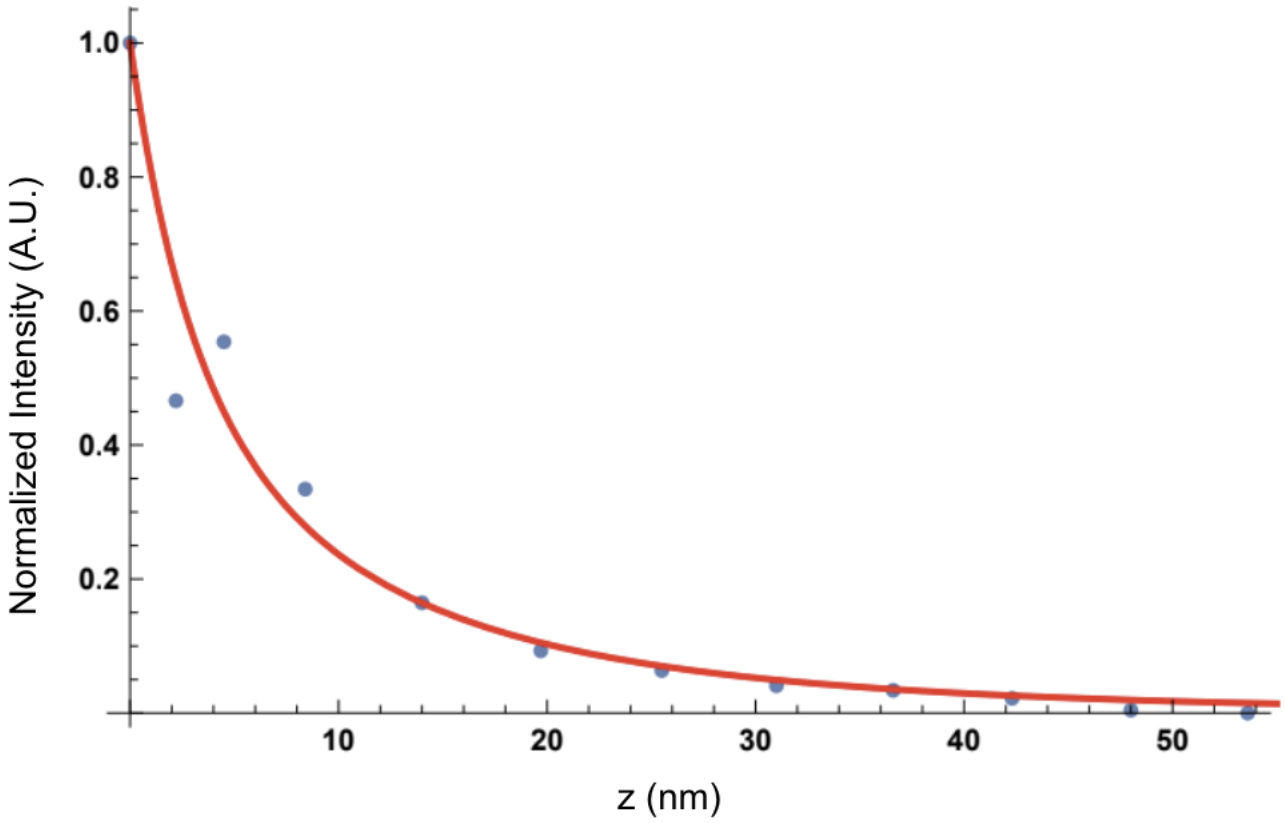

Figure 11 – Tip-approach curve fit

Fit of the graphene G' band data published in Ref. [28] and fitted using the proposed methodology.

As shown in Fig.11, the value f_e resulted from the fitting was 4.06, the same value reported in the article.